

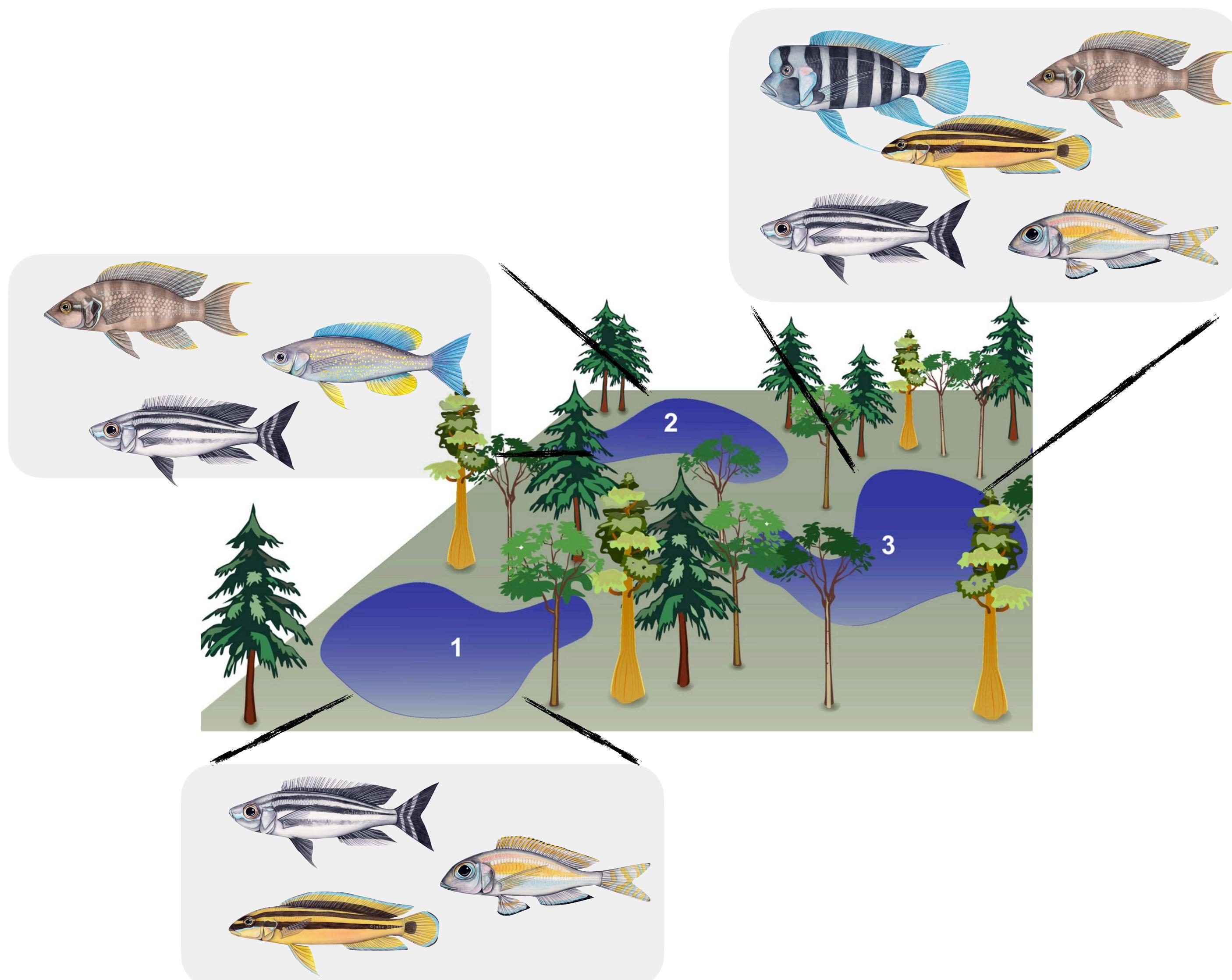
Reconceptualizing beta diversity

a hypervolume geometric approach

Chuliang Song*, Muyang Lu*,
Joseph Bennett, Benjamin Gilbert,
Marie-Josée Fortin, Andrew Gonzalez



Biological diversity over a region



α (local) diversity :=
average richness of a patch

$$\alpha = \frac{3 + 3 + 5}{3} = 3.67$$

γ (regional) diversity :=
total richness across patches

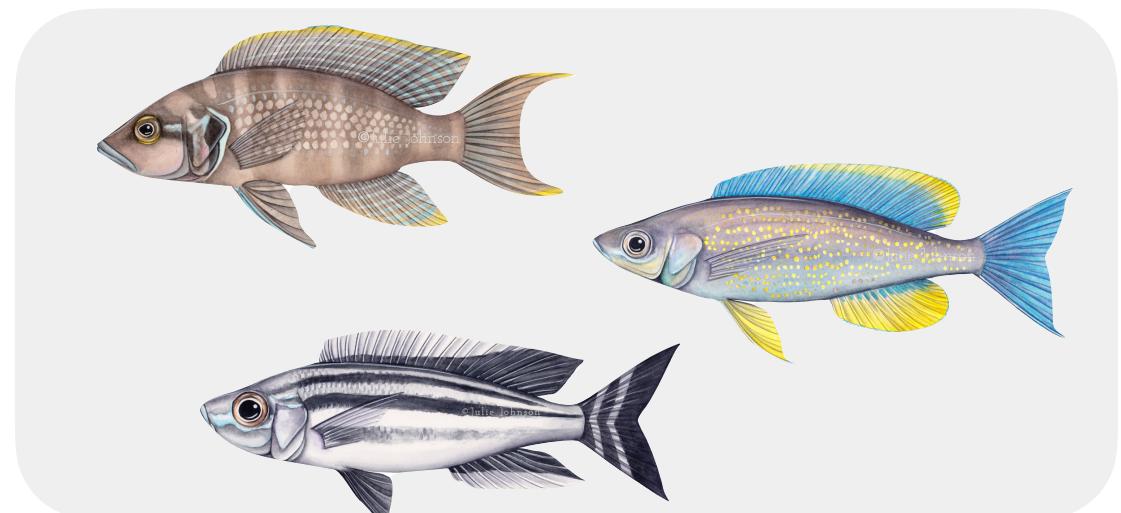
$$\gamma = 6$$

Beta diversity is a bridge from local to regional scale

Patch 1



Patch 2



Patch 3



α (local) diversity :=
average richness of a patch

β diversity

Variation among patch
compositions in a region



γ (regional) diversity :=
total richness across patches

Multiplicative beta β_{trad} : the most used definition in the literature

$$\beta_{\text{trad}} := \frac{\gamma}{\alpha}$$

Metacommunity I



$$\beta_{\text{trad}}(I) = \frac{\gamma}{\alpha} = \frac{2}{1}$$

Metacommunity II

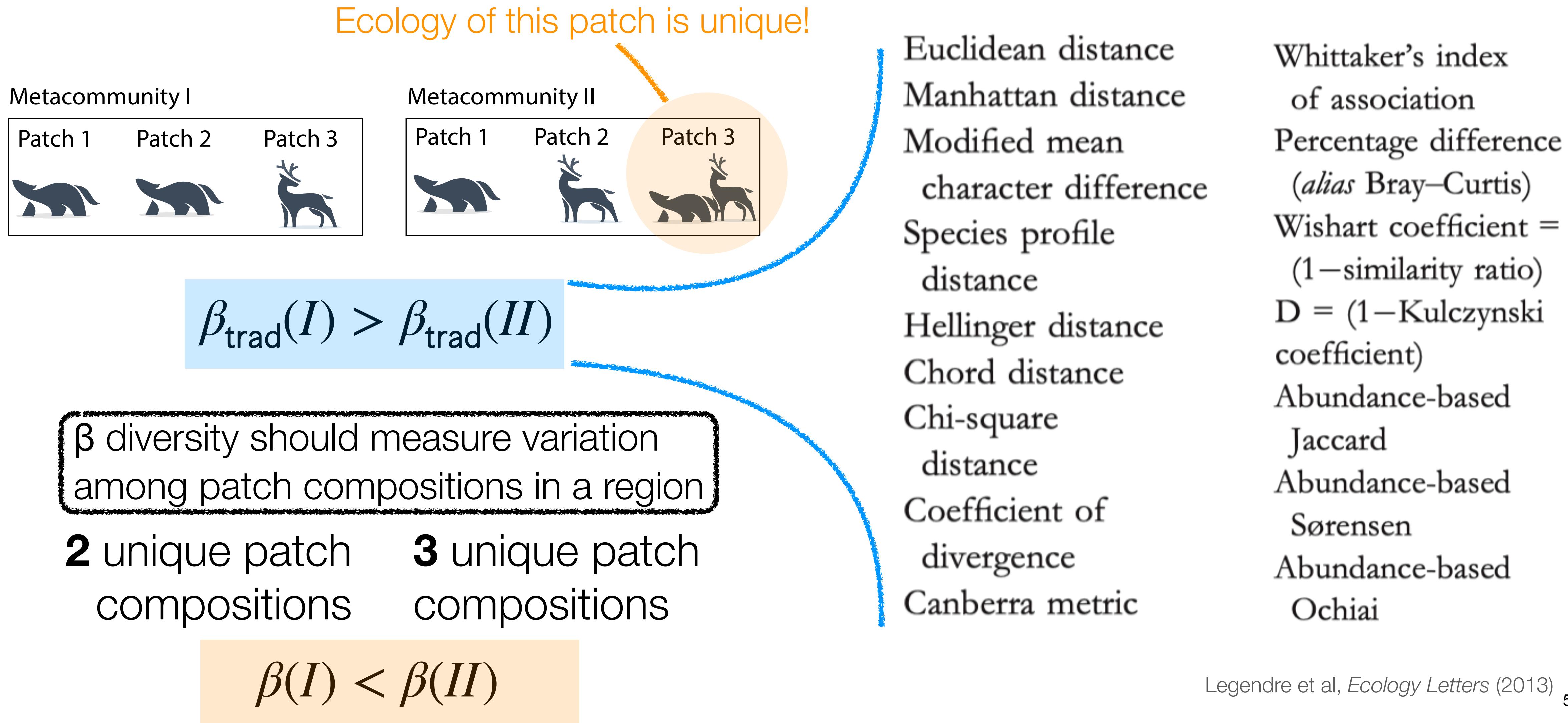


$$\beta_{\text{trad}}(II) = \frac{\gamma}{\alpha} = \frac{2}{1.33}$$

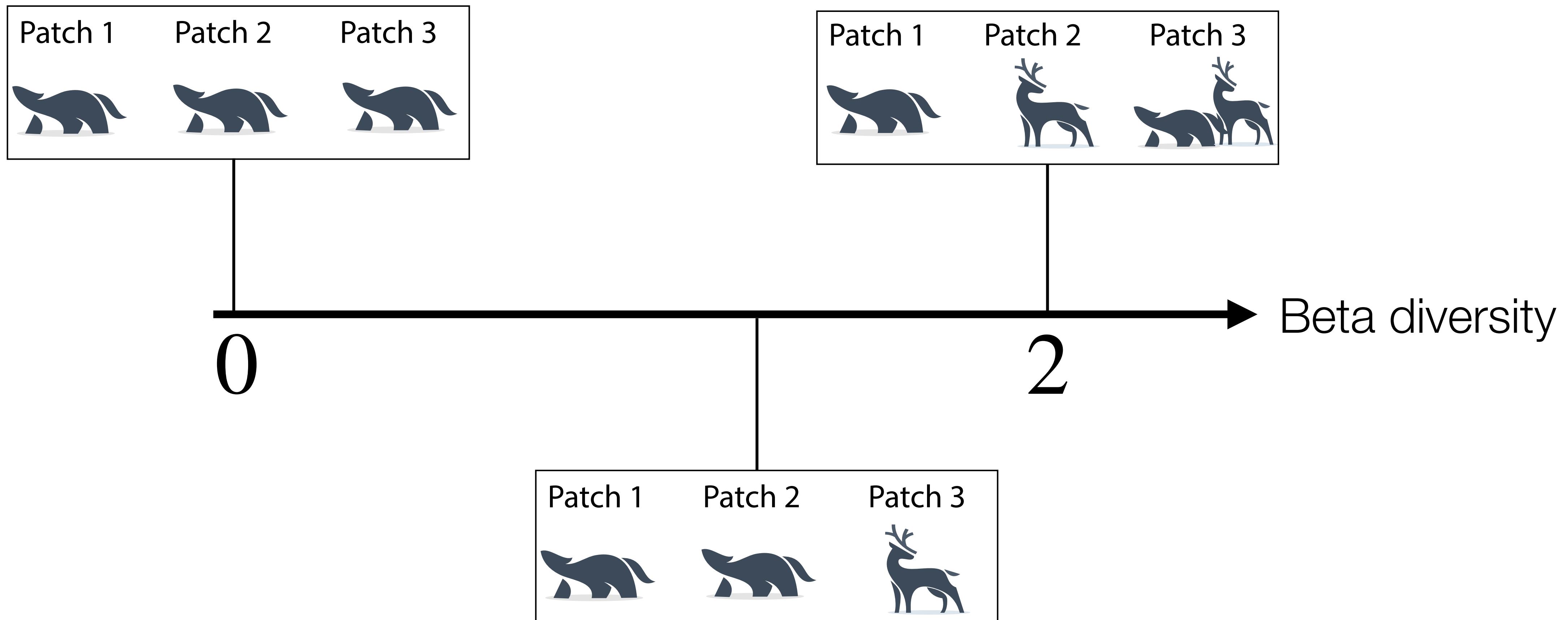
$$\beta_{\text{trad}}(I) > \beta_{\text{trad}}(II)$$

Axiom satisfied by all traditional measures of beta diversity:

Beta diversity is maximized when each patch has one distinct species



Goal: we want to design a measure of beta diversity that does this



A geometric approach to beta diversity

A geometric approach to beta diversity

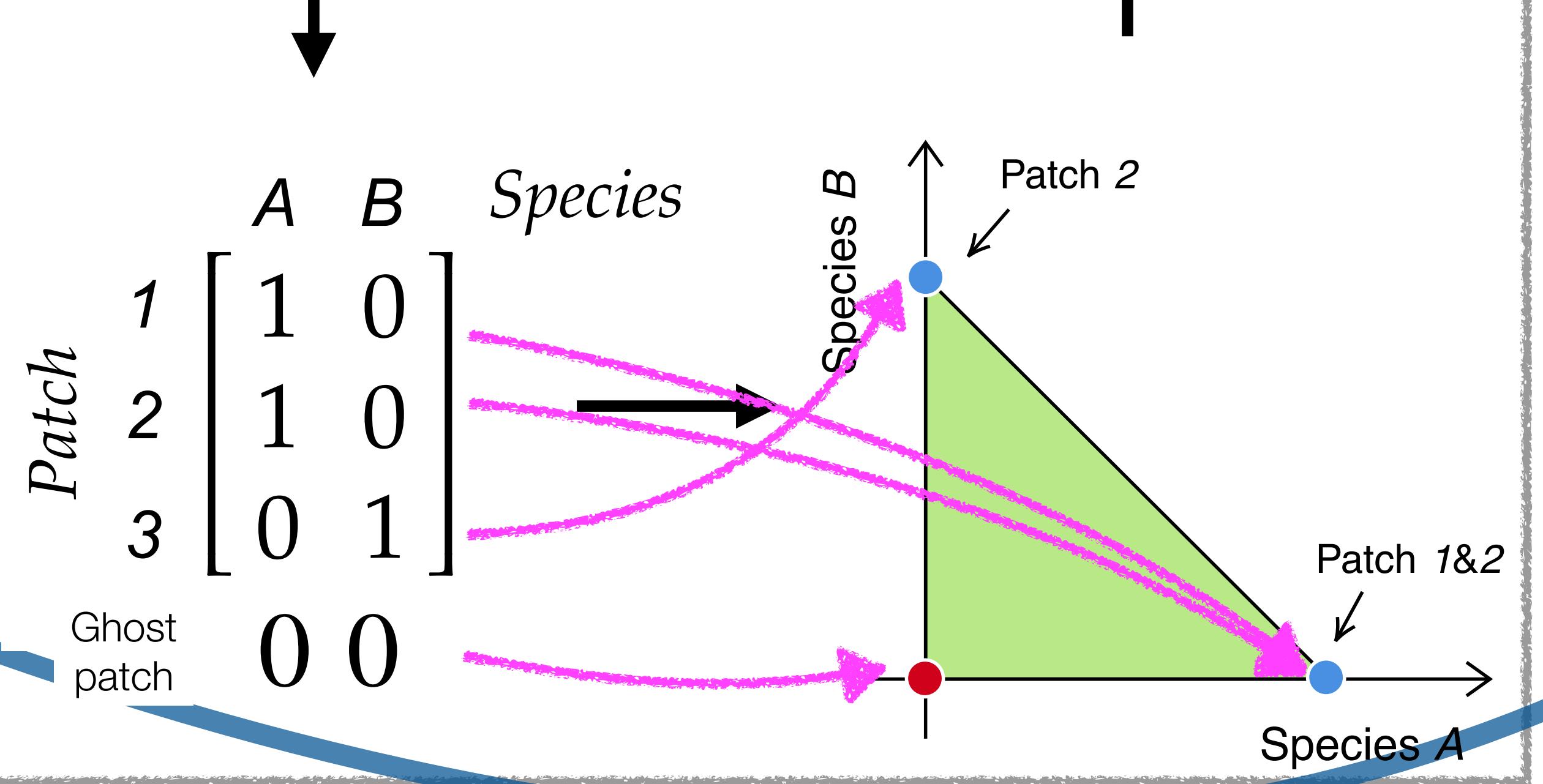
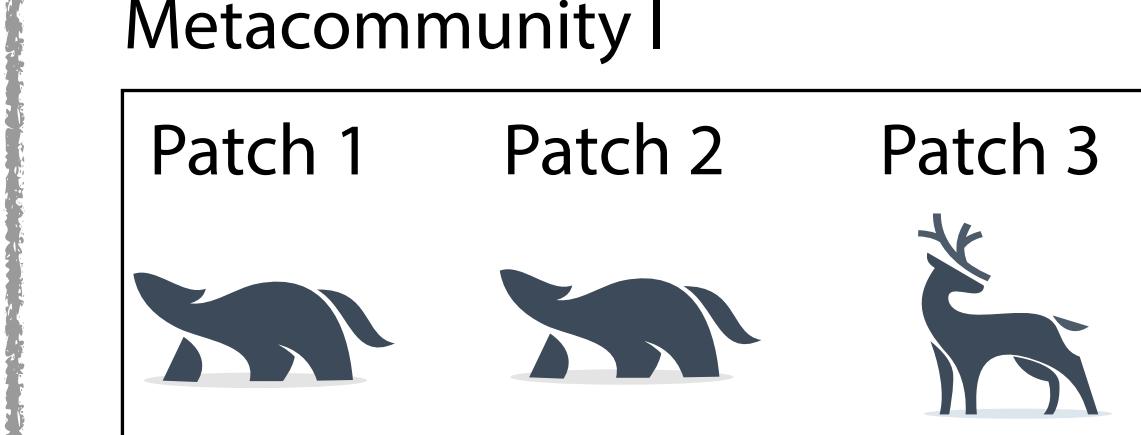
Metacommunity composition



Metacommunity matrix



Example



$$\beta_{\text{vol}} = 2 \times \left(\frac{1}{2}\right)^{1/2} = 1.4$$

Embedded dimension

$$\beta_{\text{vol}} := d \times (\text{vol}(P))^{1/d}$$

0 ≤ Hypervolume ≤ d

Beta diversity as hypervolume

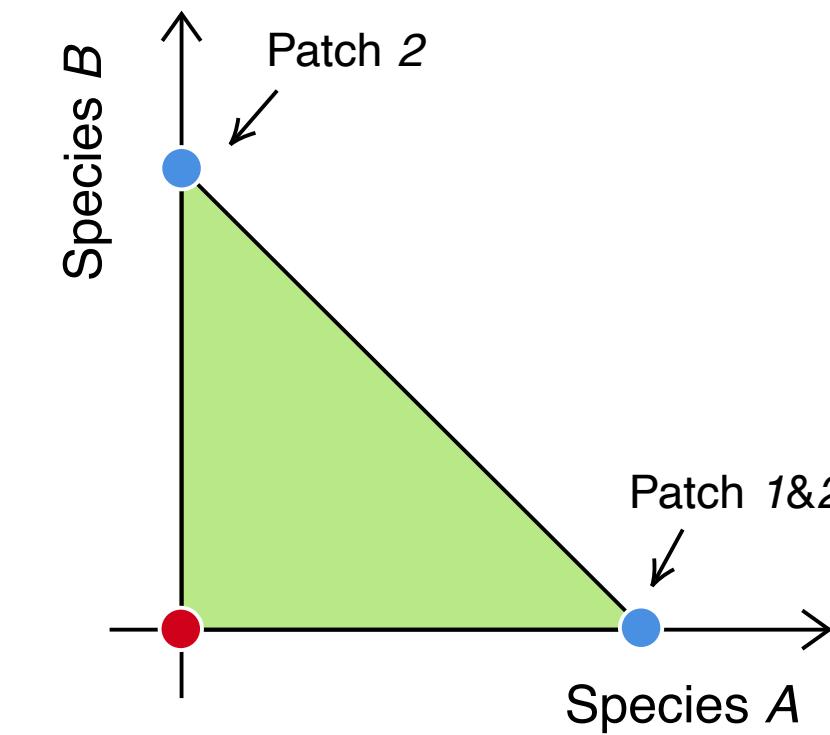
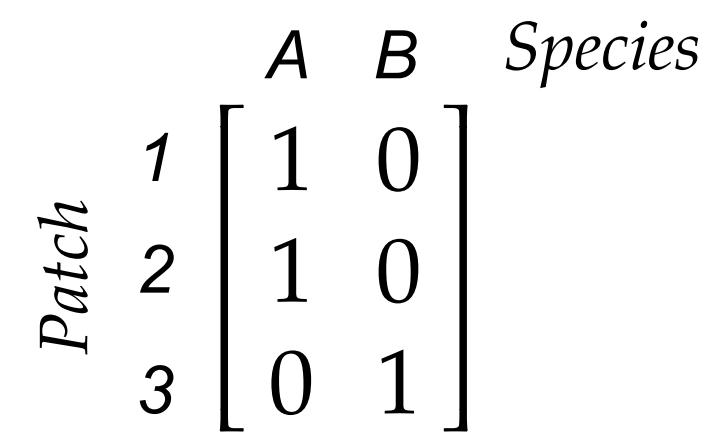
Rescaling of hypervolume

$$\text{Hypervolume} \leq d$$

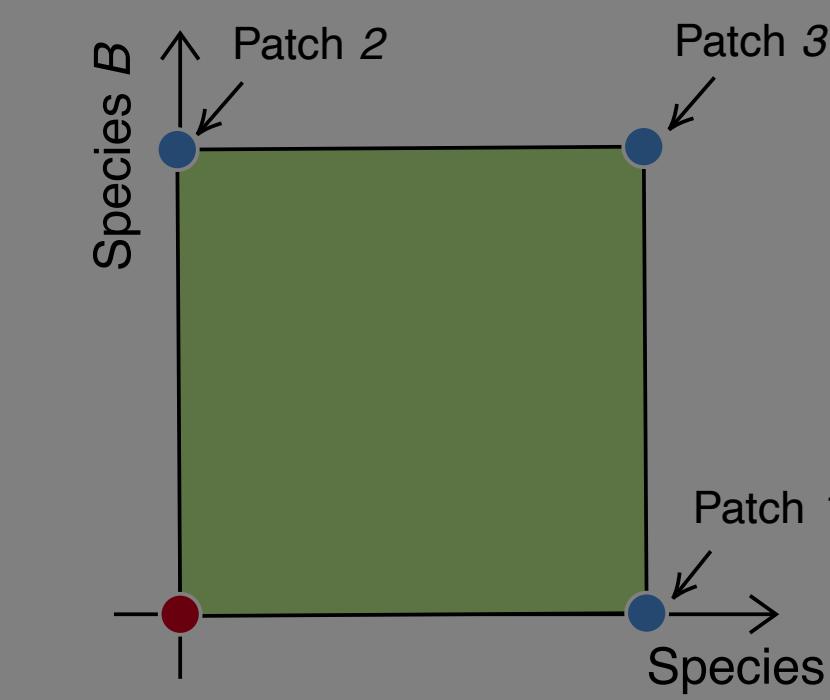
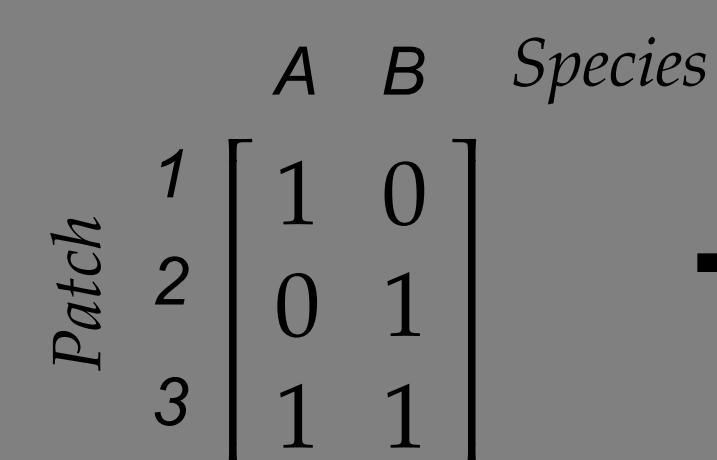
Geometric embedding

β_{vol} for metacommunities with 2 species

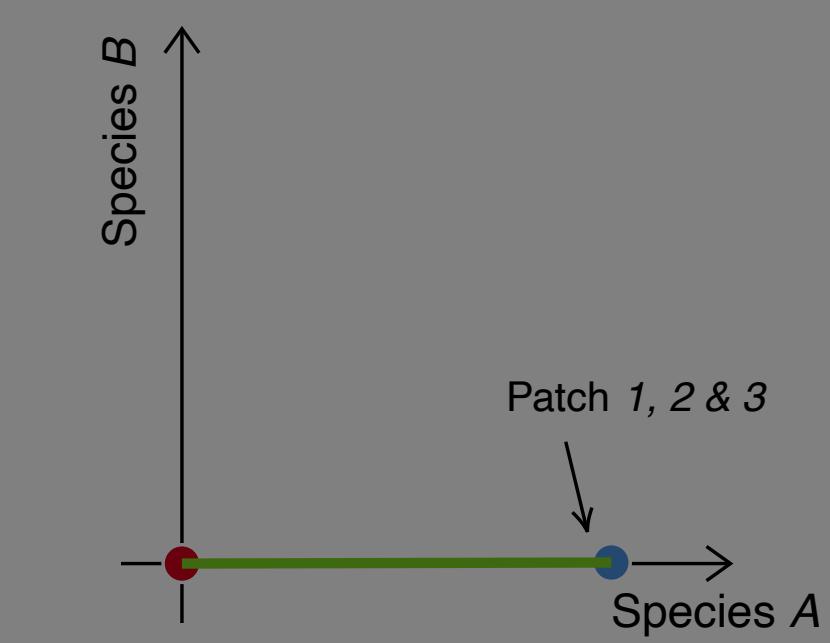
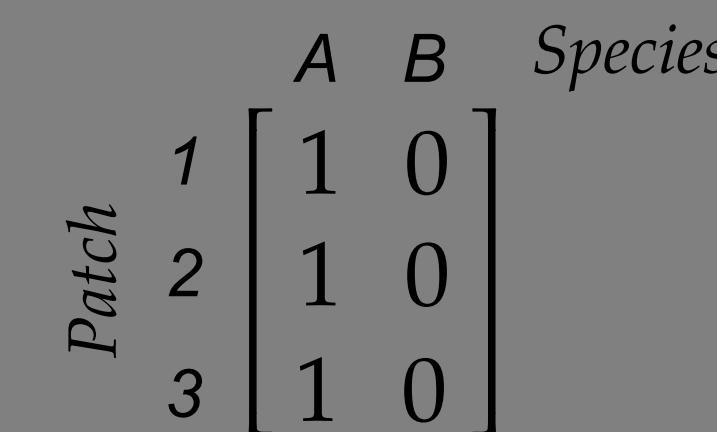
$$\beta_{\text{vol}} := d \times (\text{vol}(P))^{1/d}$$



$$\beta_{\text{vol}} = 2 \times \left(\frac{1}{2}\right)^{1/2} = 1.4$$



$$\beta_{\text{vol}} = 2 \times (1)^{1/2} = 2$$



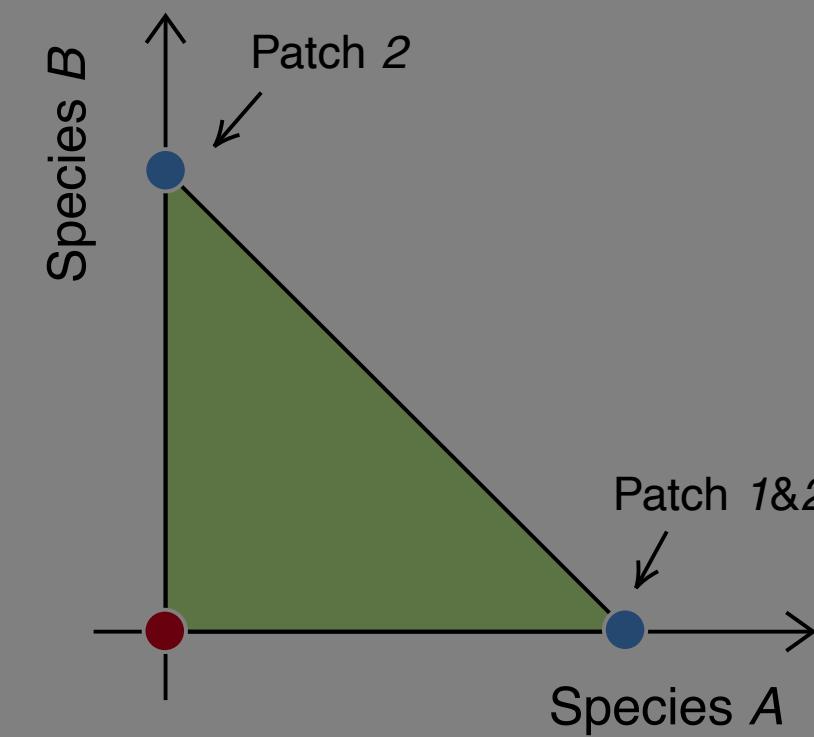
$$\beta_{\text{vol}} = 2 \times (0)^{1/2} = 0$$

β_{vol} for metacommunities with 2 species

$$\beta_{\text{vol}} := d \times (\text{vol}(P))^{1/d}$$



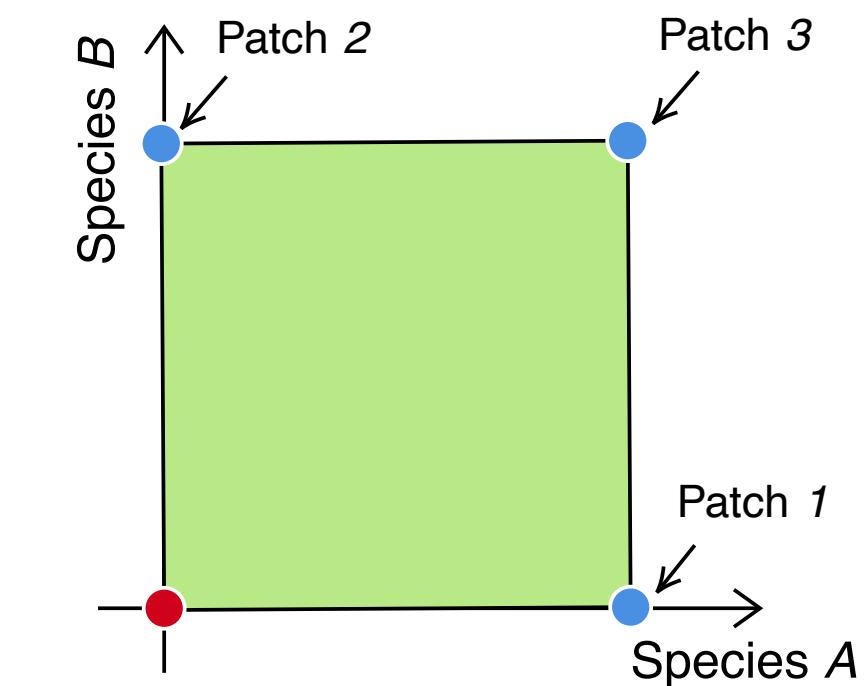
	A	B	Species
Patch 1	1	0	
Patch 2	1	0	
Patch 3	0	1	



$$\beta_{\text{vol}} = 2 \times \left(\frac{1}{2}\right)^{1/2} = 1.4$$



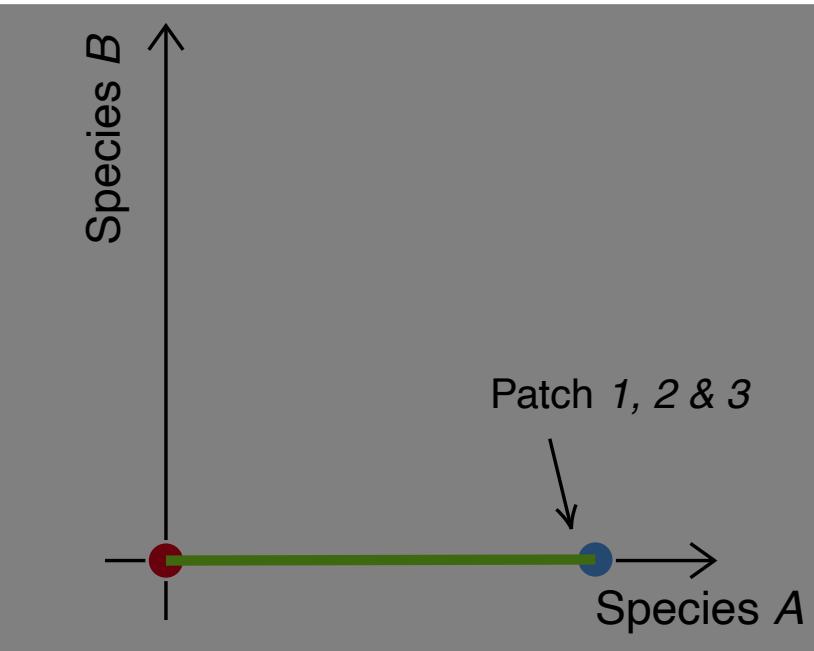
	A	B	Species
Patch 1	1	0	
Patch 2	0	1	
Patch 3	1	1	



$$\beta_{\text{vol}} = 2 \times (1)^{1/2} = 2$$



	A	B	Species
Patch 1	1	0	
Patch 2	1	0	
Patch 3	1	0	



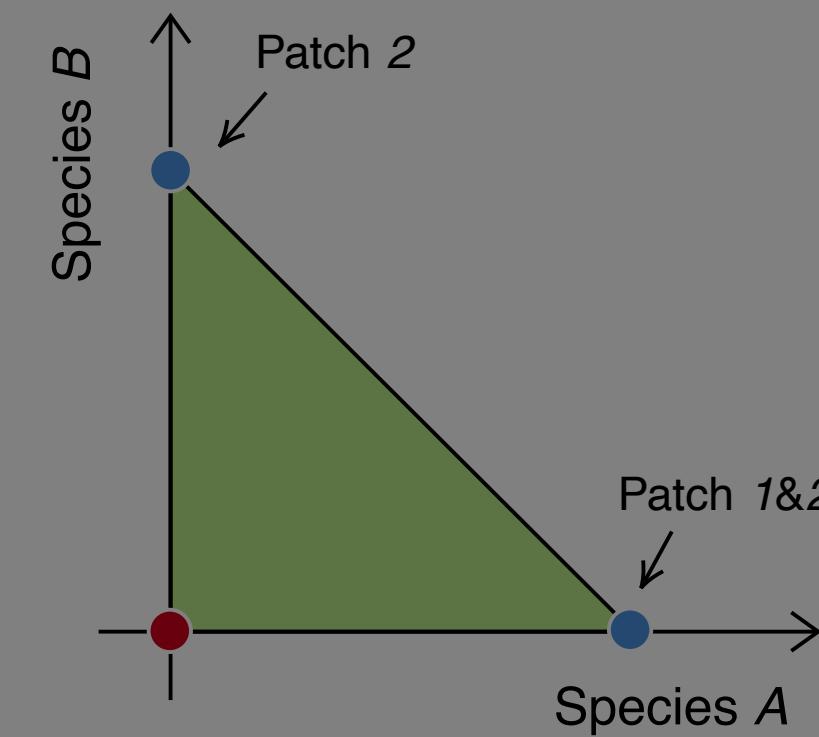
$$\beta_{\text{vol}} = 2 \times (0)^{1/2} = 0$$

β_{vol} for metacommunities with 2 species

$$\beta_{\text{vol}} := d \times (\text{vol}(P))^{1/d}$$



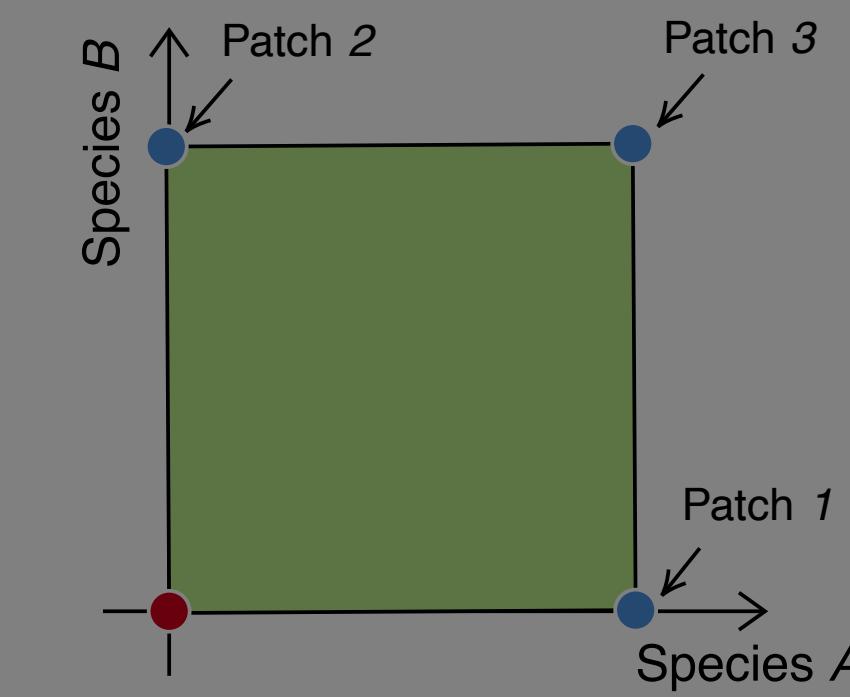
	Species	
Patch	A	B
1	1	0
2	1	0
3	0	1



$$\beta_{\text{vol}} = 2 \times \left(\frac{1}{2}\right)^{1/2} = 1.4$$



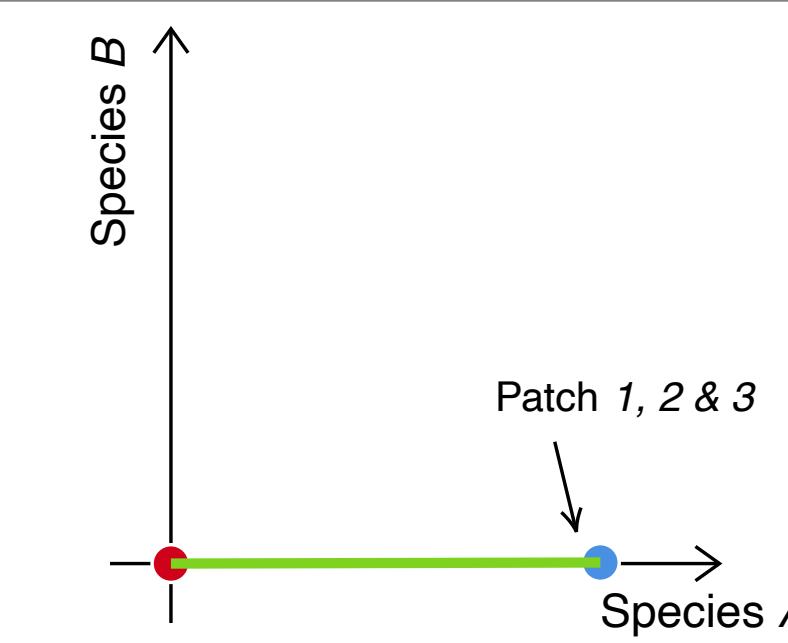
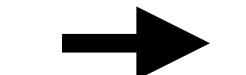
	Species	
Patch	A	B
1	1	0
2	0	1
3	1	1



$$\beta_{\text{vol}} = 2 \times (1)^{1/2} = 2$$

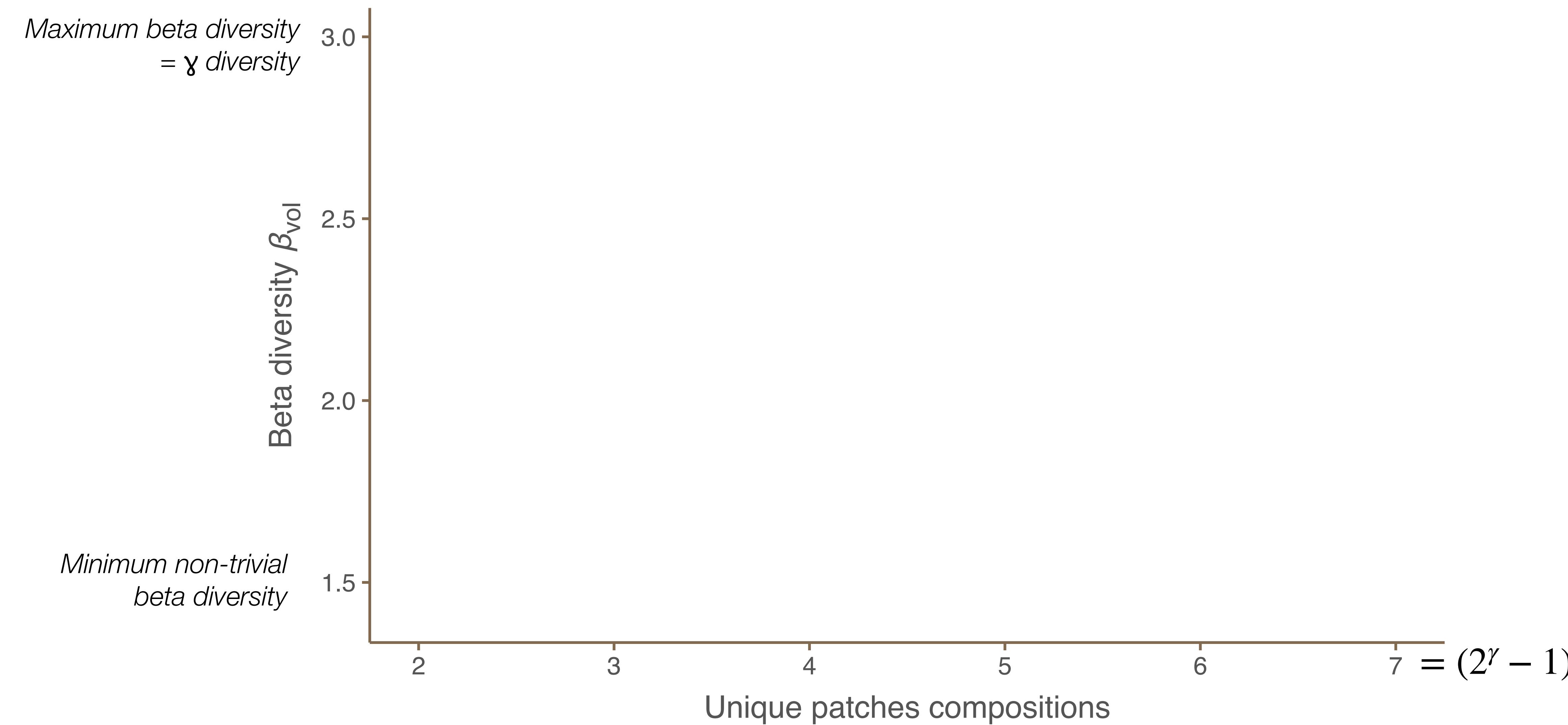


	Species	
Patch	A	B
1	1	0
2	1	0
3	1	0

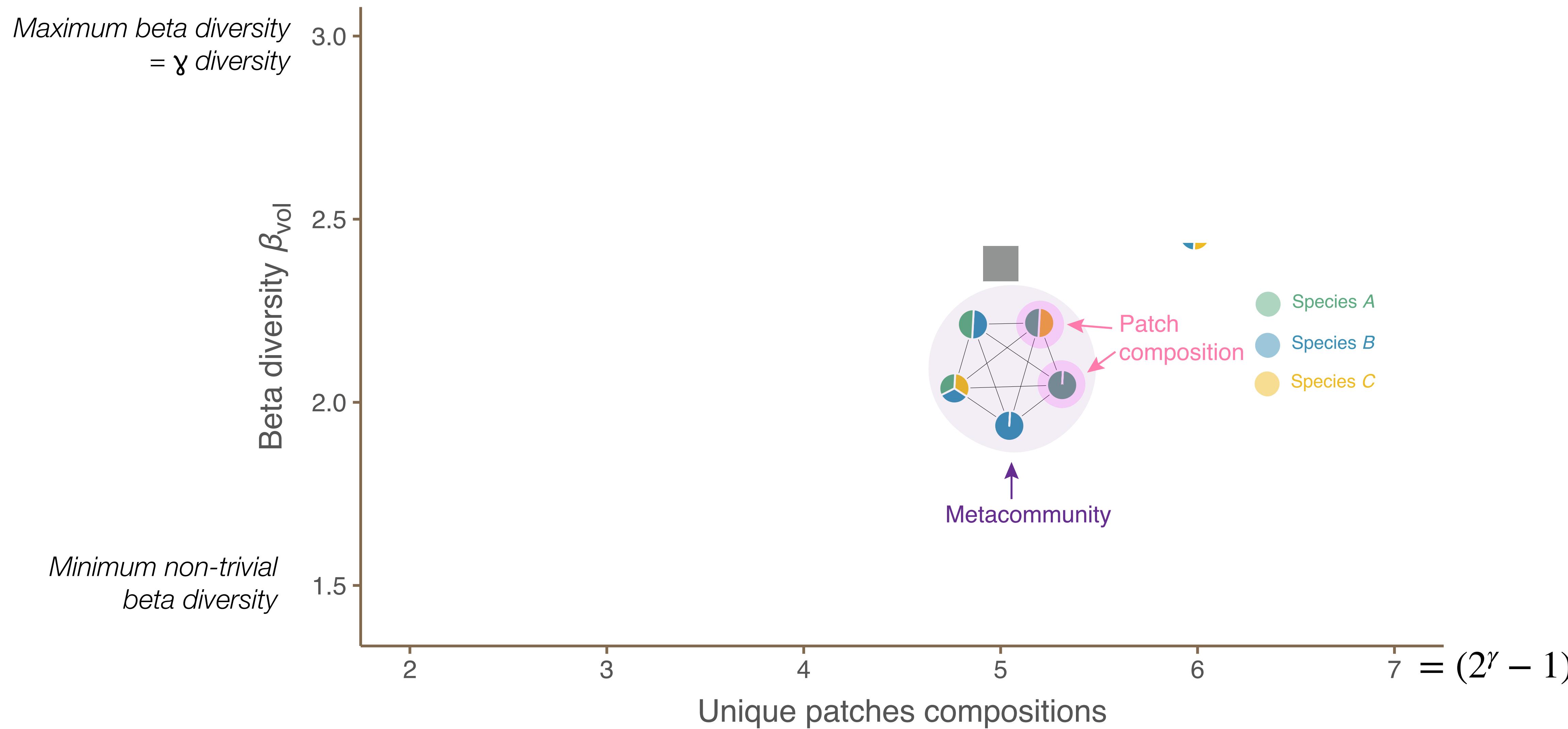


$$\beta_{\text{vol}} = 2 \times (0)^{1/2} = 0$$

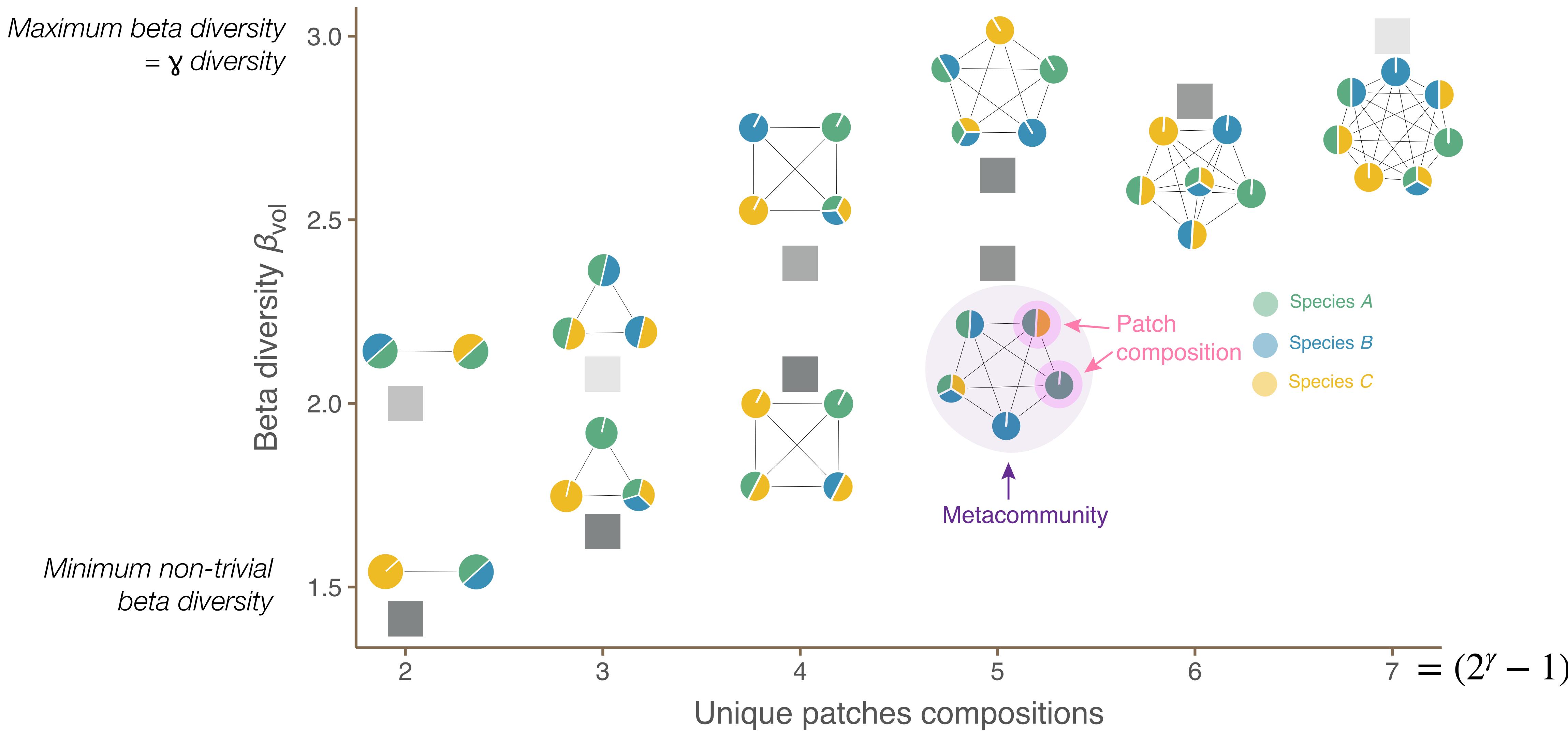
Hypervolume beta β_{vol} for all metacommunities with $\gamma = 3$ species



Hypervolume beta β_{vol} for all metacommunities with $\gamma = 3$ species



Hypervolume beta β_{vol} for all metacommunities with $\gamma = 3$ species



Temporal change
of beta diversity

Patch-level contribution
to beta diversity

Species similarity and
functional complementarity

A unified framework of beta diversity

Duplications in
presence/absence data

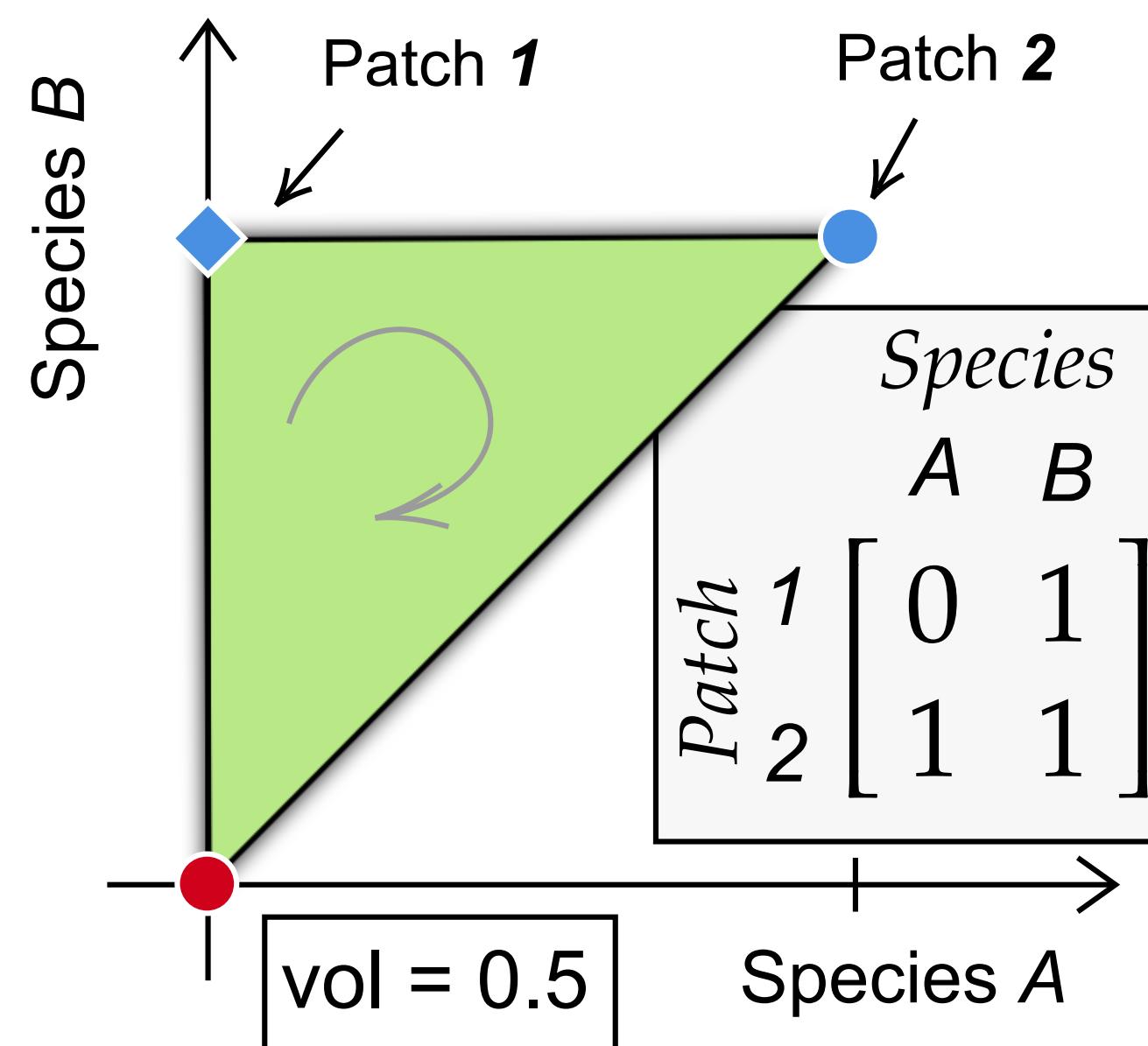
Nestedness-turnover
decomposition

Temporal change of beta diversity

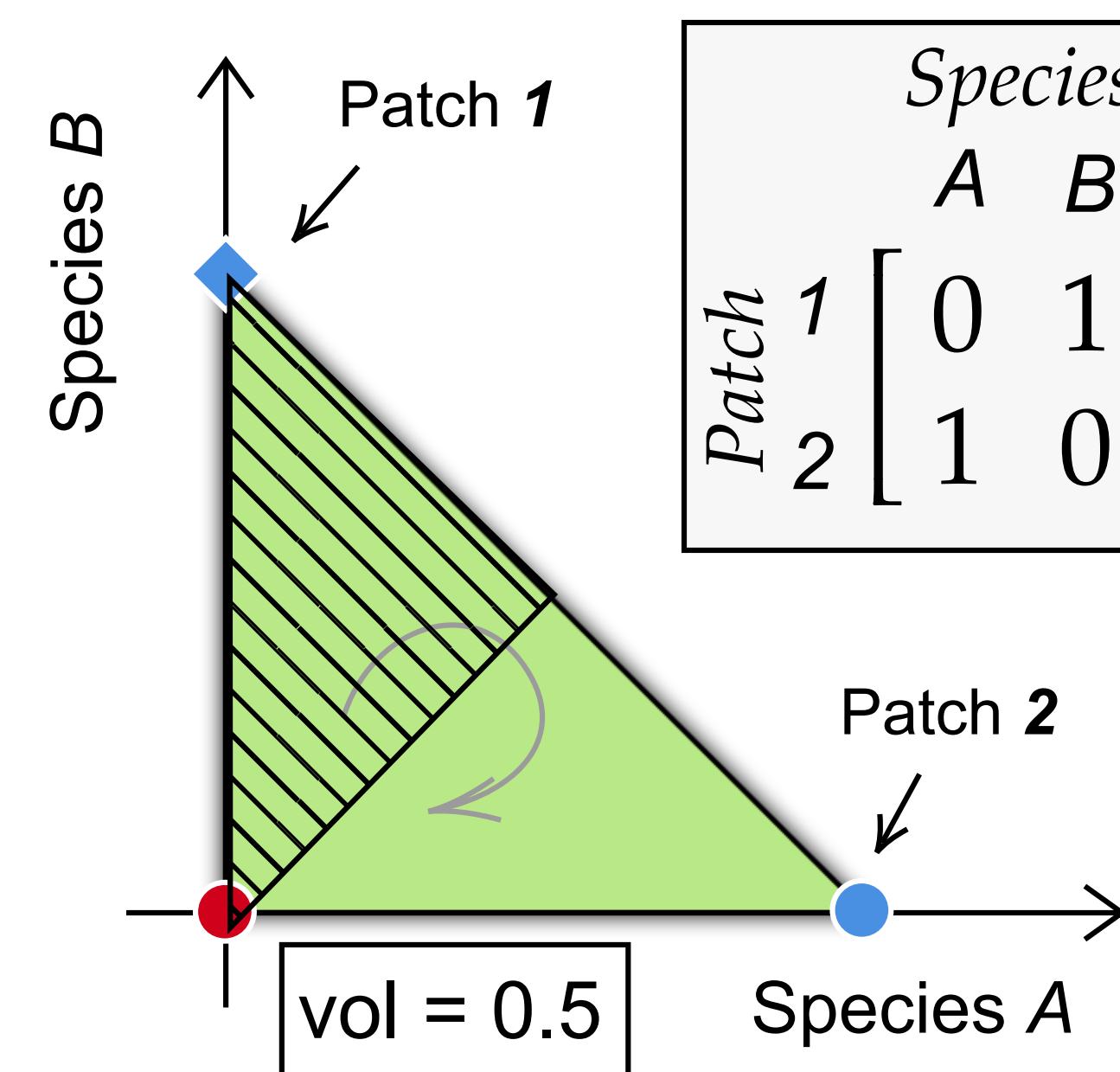
Orientation of hypervolume (*synchronization direction*):

Patch 1 → Patch 2 → Origin → Patch 1

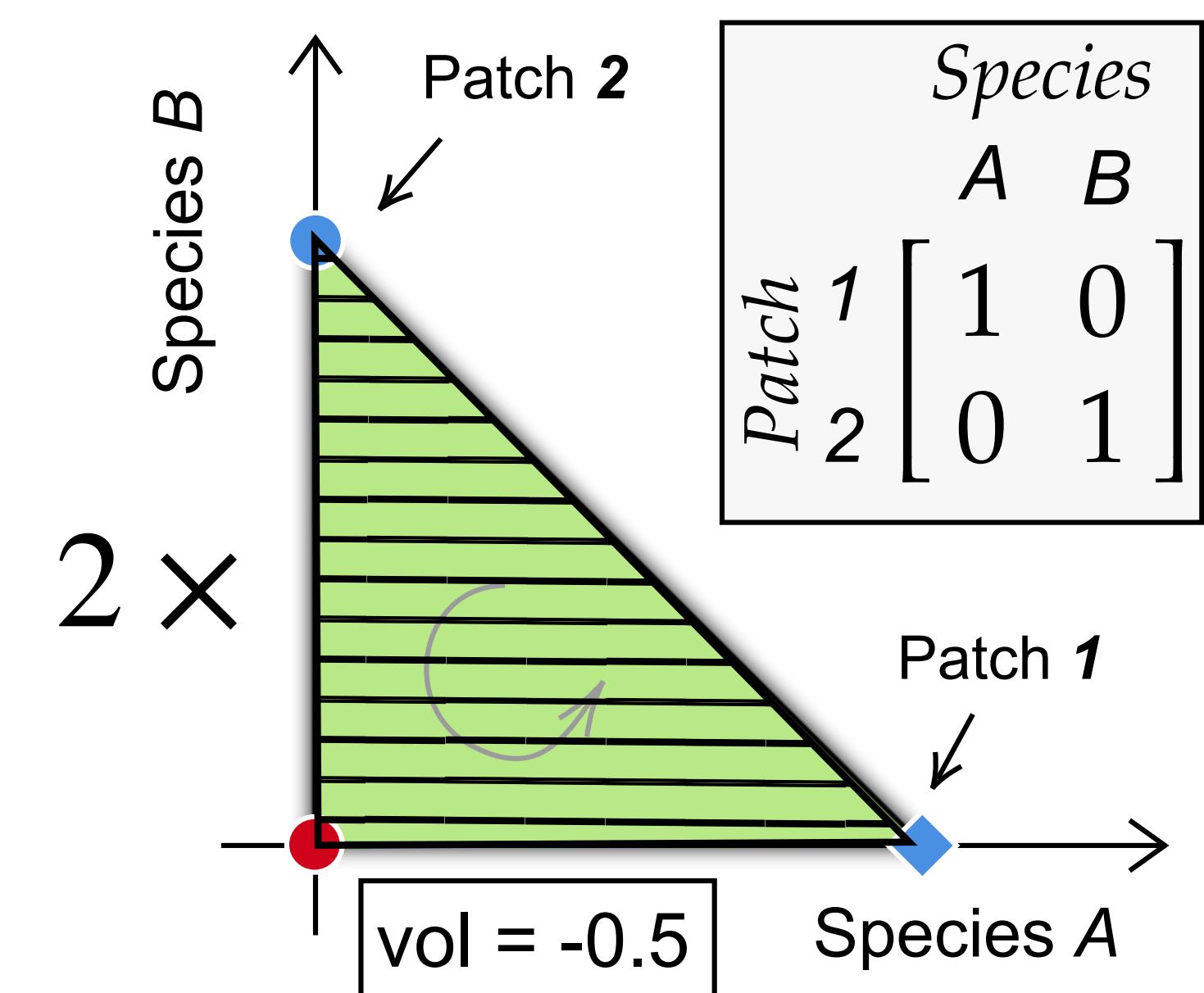
A. time = t



B. time = $t + 1$



C. time = $t + 2$

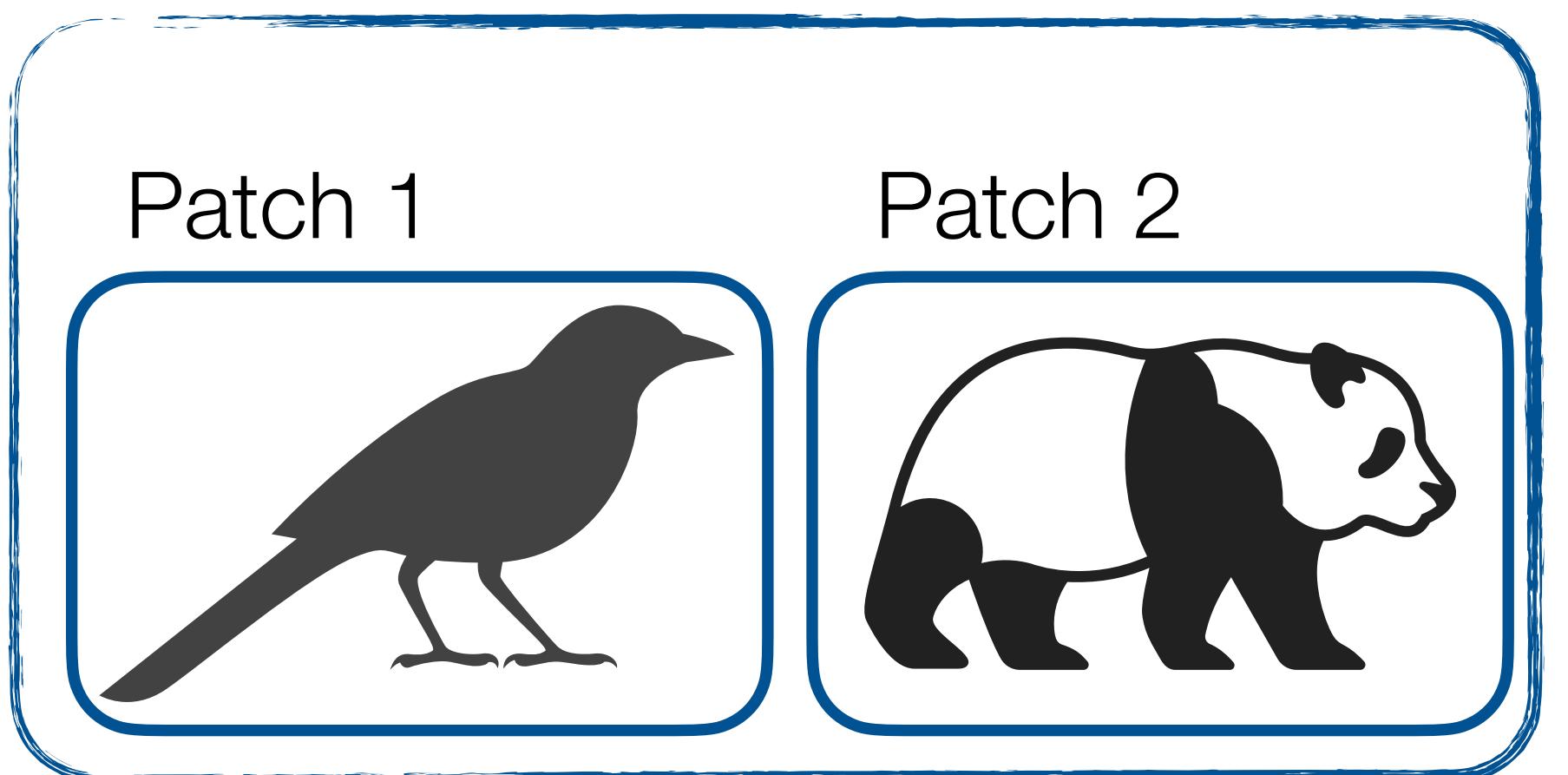


$$\beta_{t \rightarrow (t+1)} = d \times \text{vol}(P_{t+1} \cap P_1)^{1/d} = 1$$

$$\beta_{(t+1) \rightarrow (t+2)} = 2$$

Species similarity should affect beta diversity

Metacommunity I

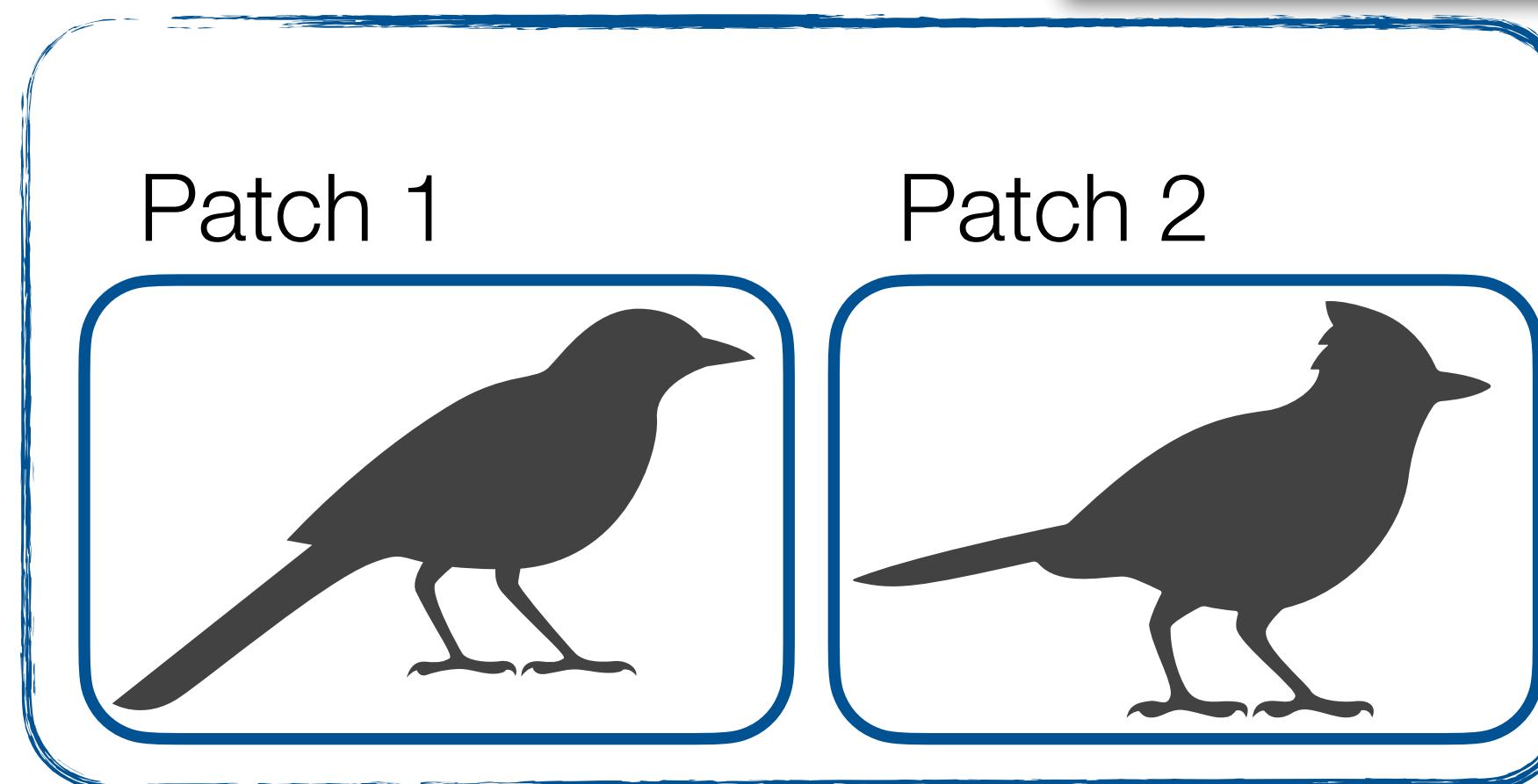


Species are very dissimilar



Higher beta diversity

Metacommunity II



Species are very similar

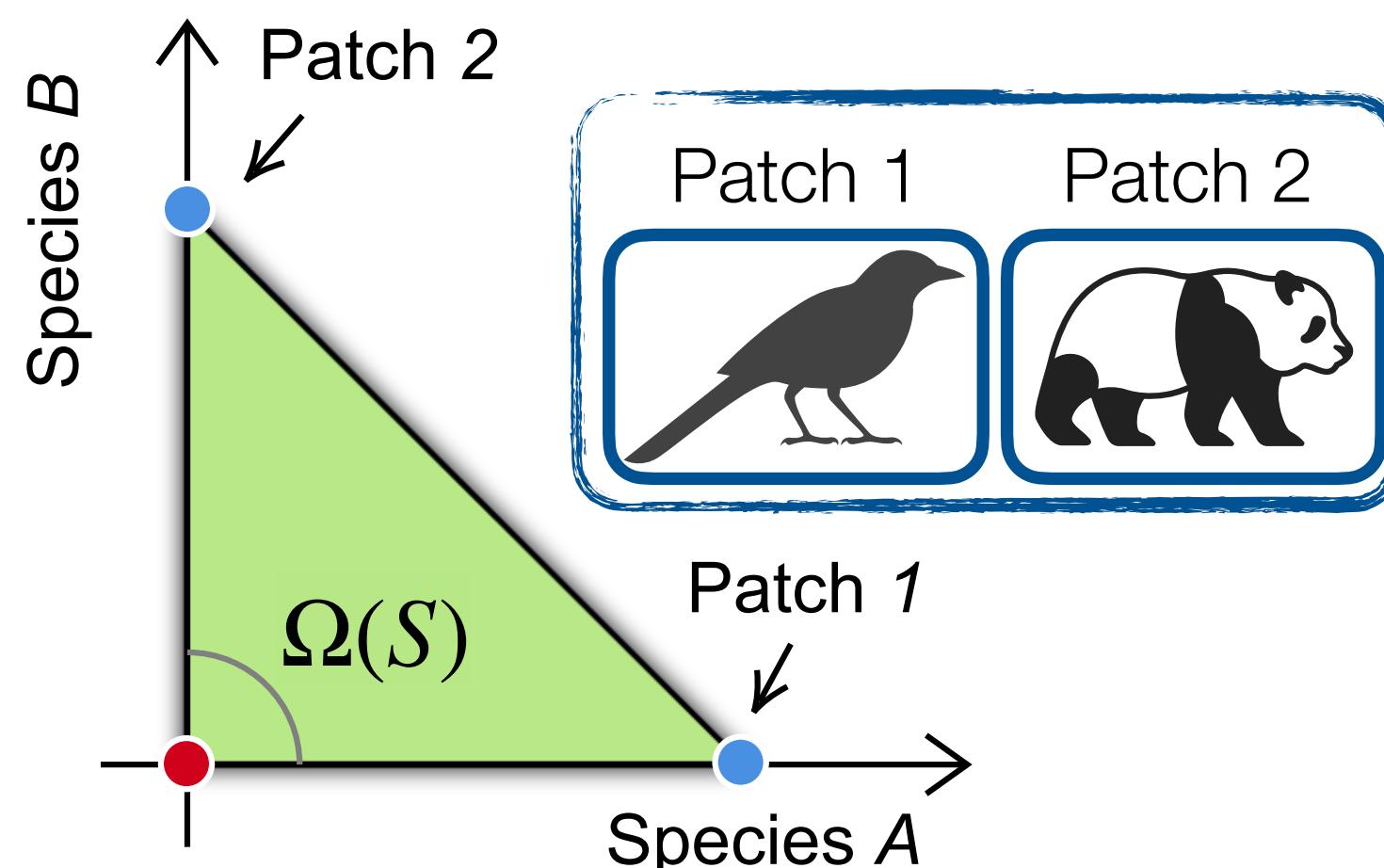


Lower beta diversity

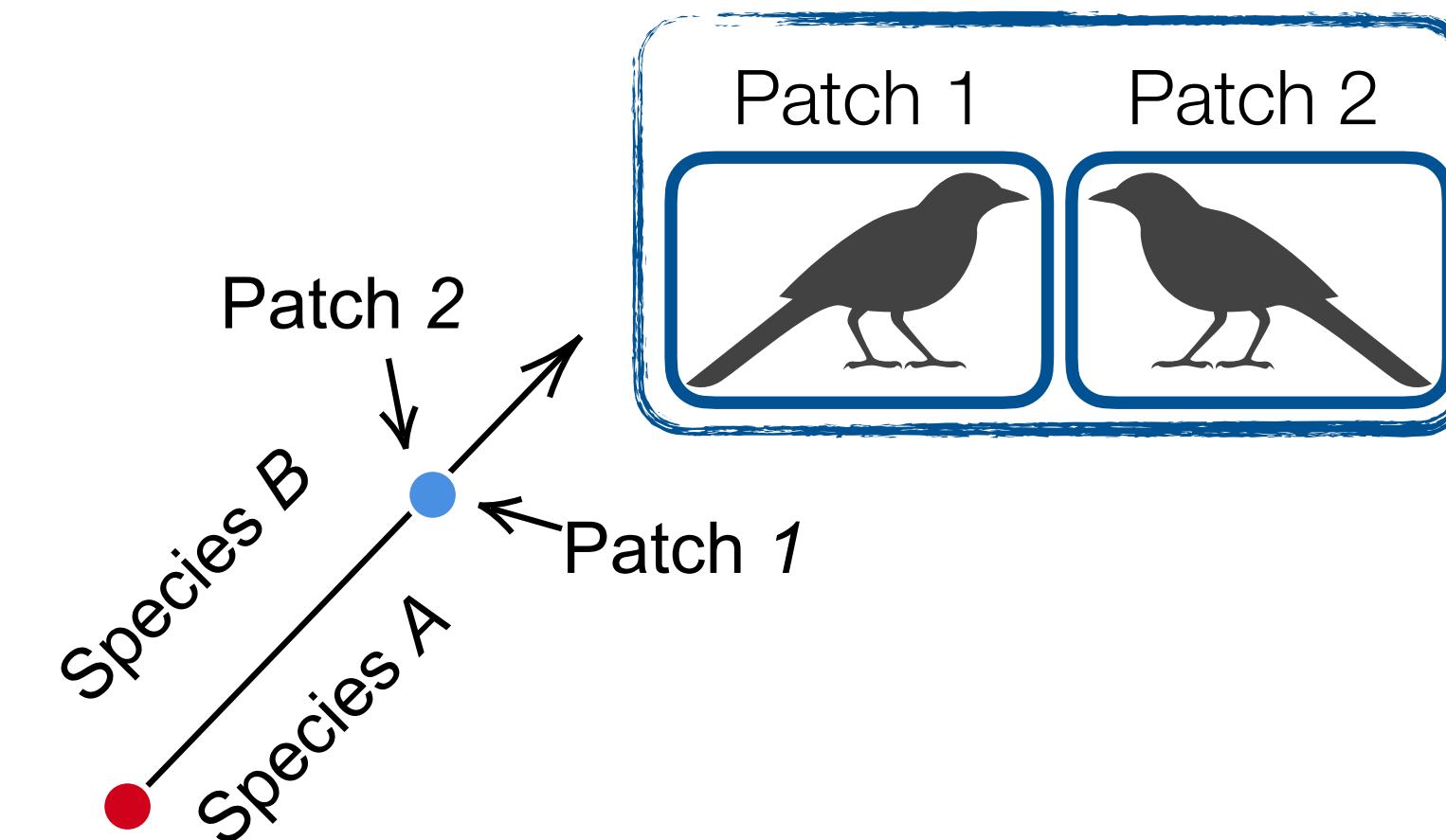


Species similarity as compressing hypervolume

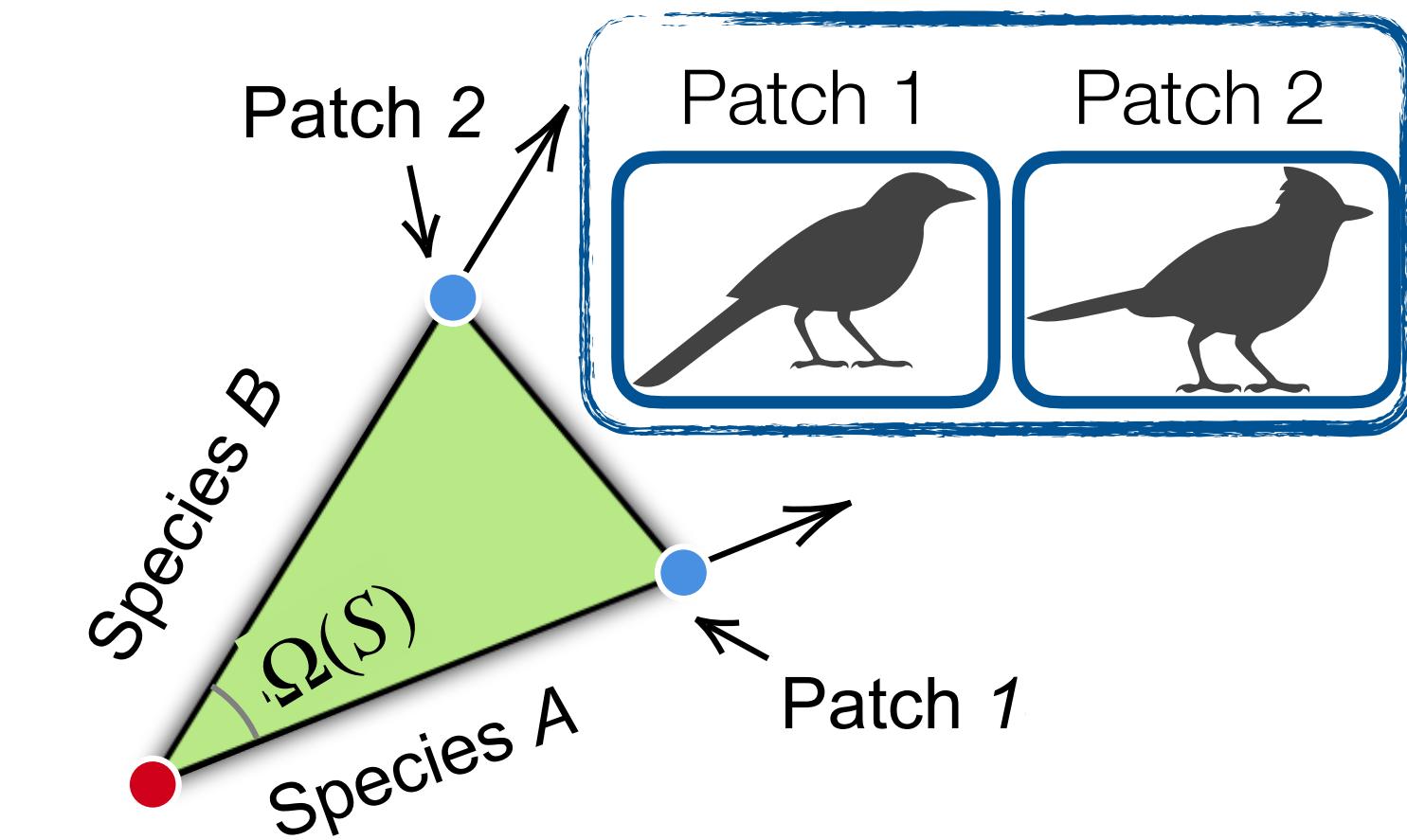
A. Species are totally dissimilar



B. Species are totally similar



C. Species are a bit similar



Similarity matrix S

$$S = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

New x axis
New y axis

- Species is always 100% similar to itself
- How similar species A is to species B

$$S = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

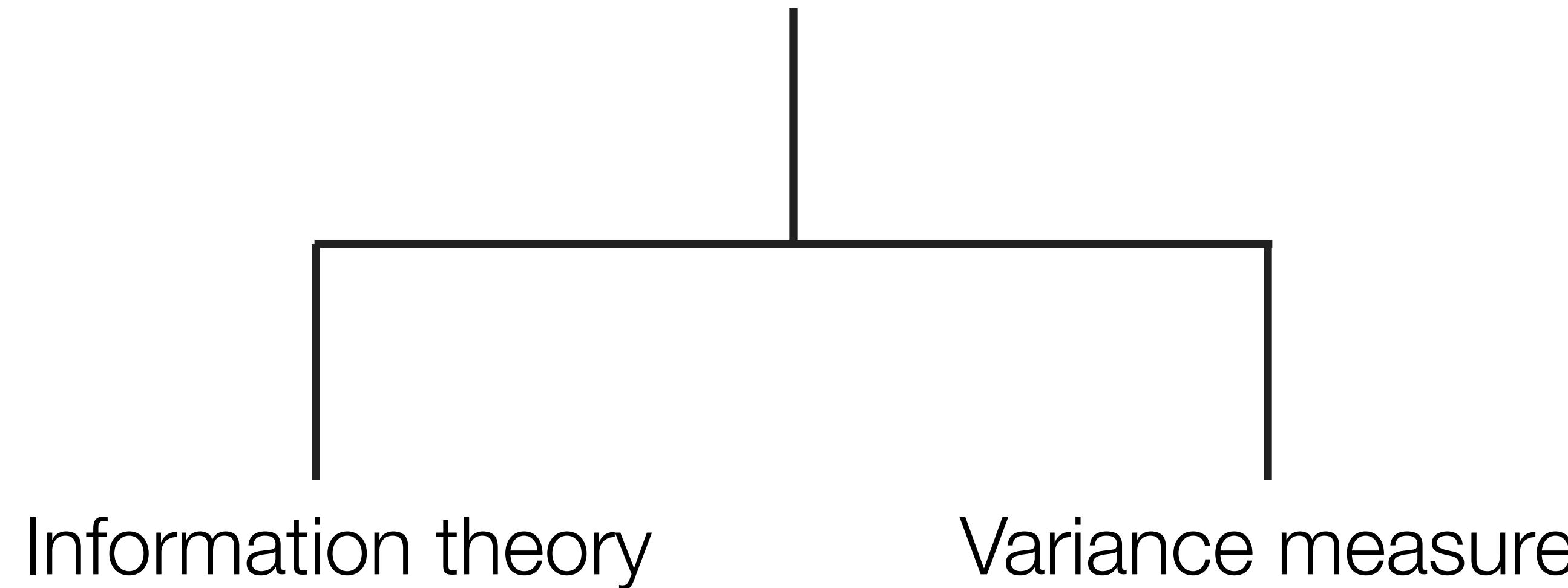
$$S = \begin{bmatrix} 1 & .5 \\ .5 & 1 \end{bmatrix}$$

Original hypervolume

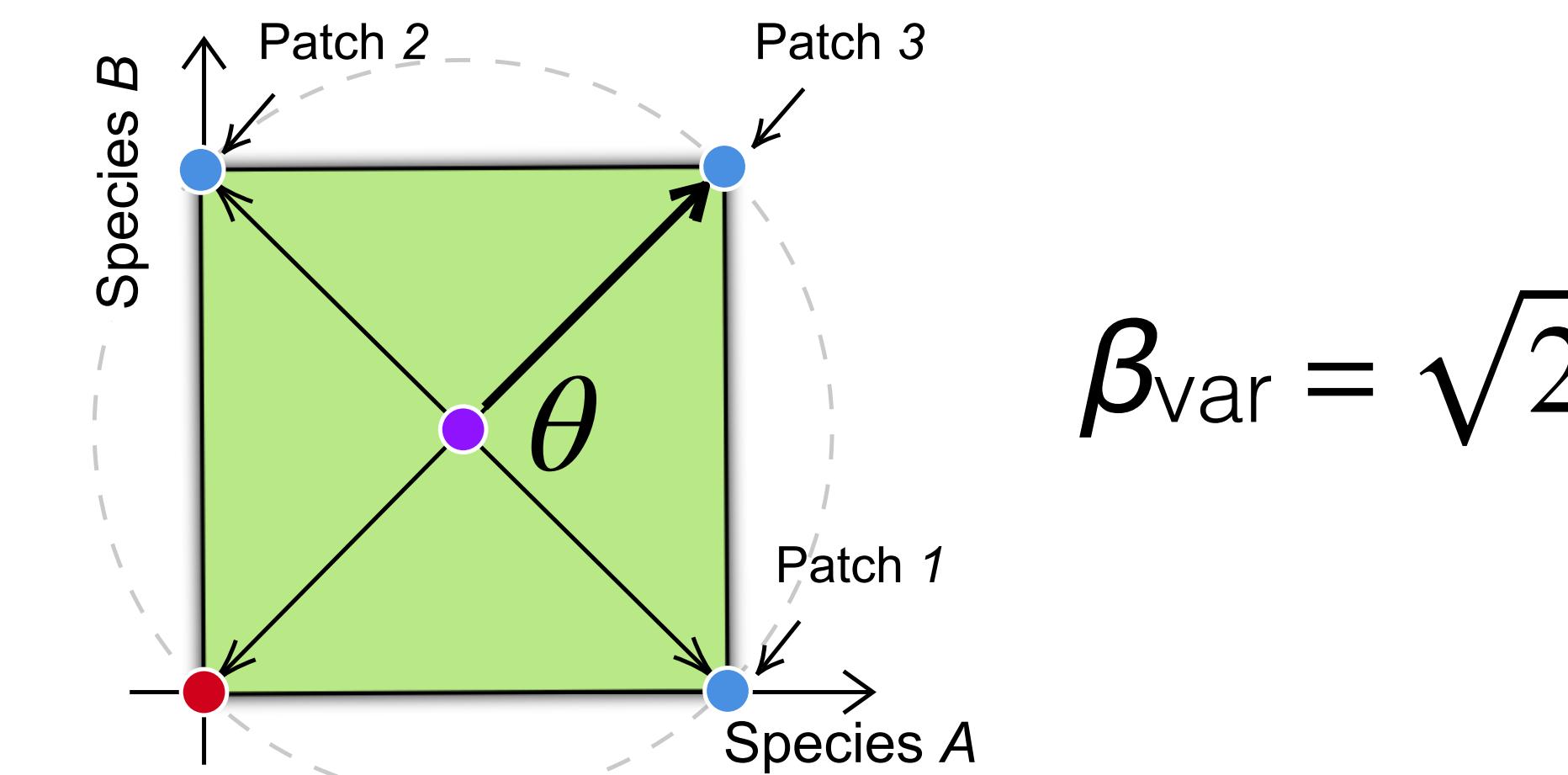
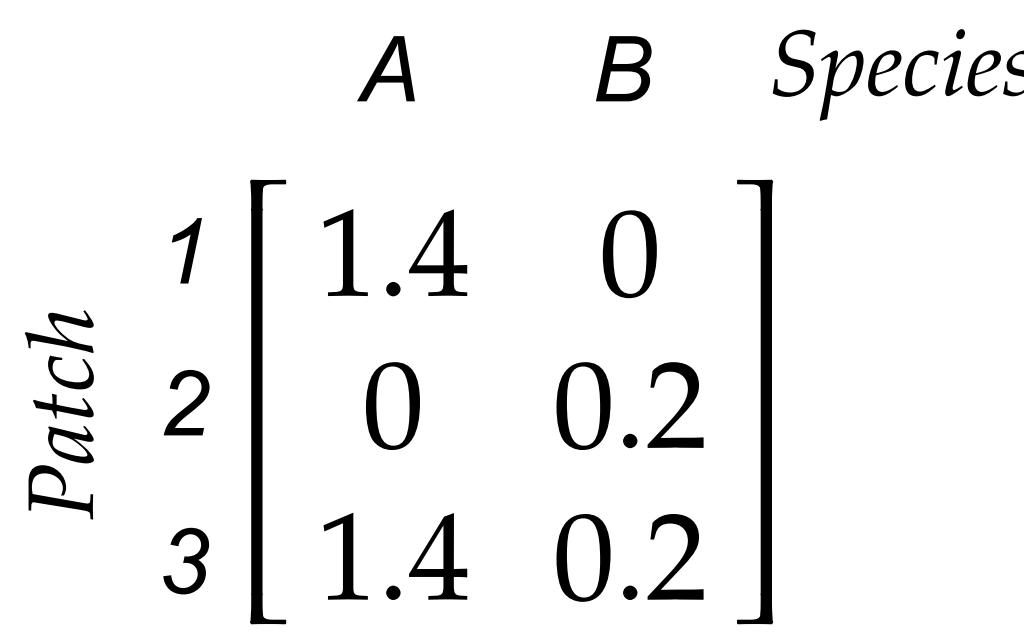
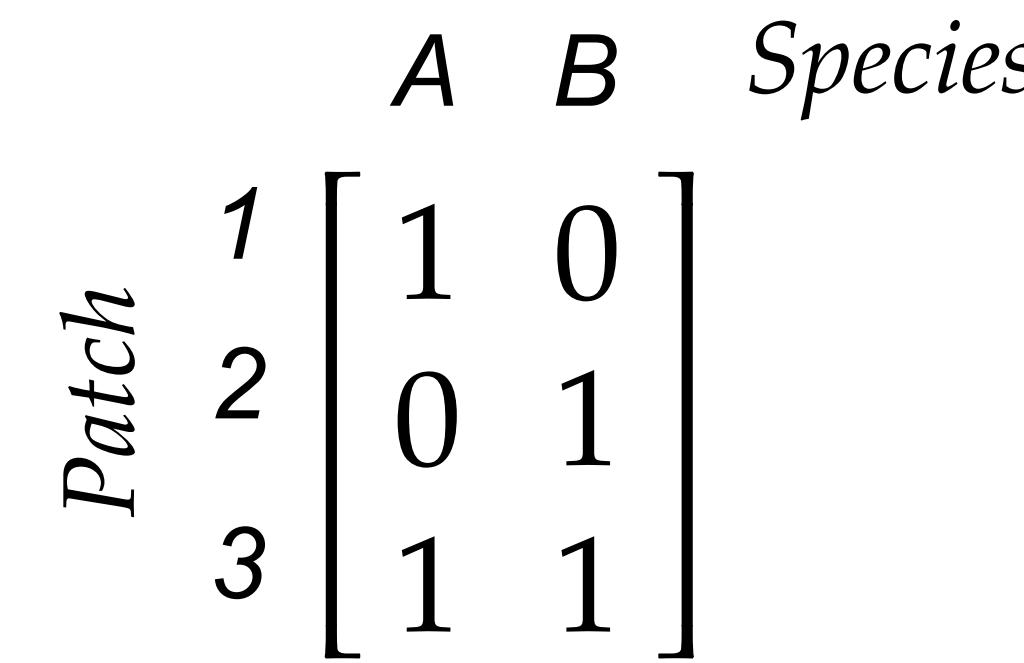
$$\beta_{\text{vol}} = d \times (\Omega(S) \text{vol}(P))^{1/d}$$

Compressed angle

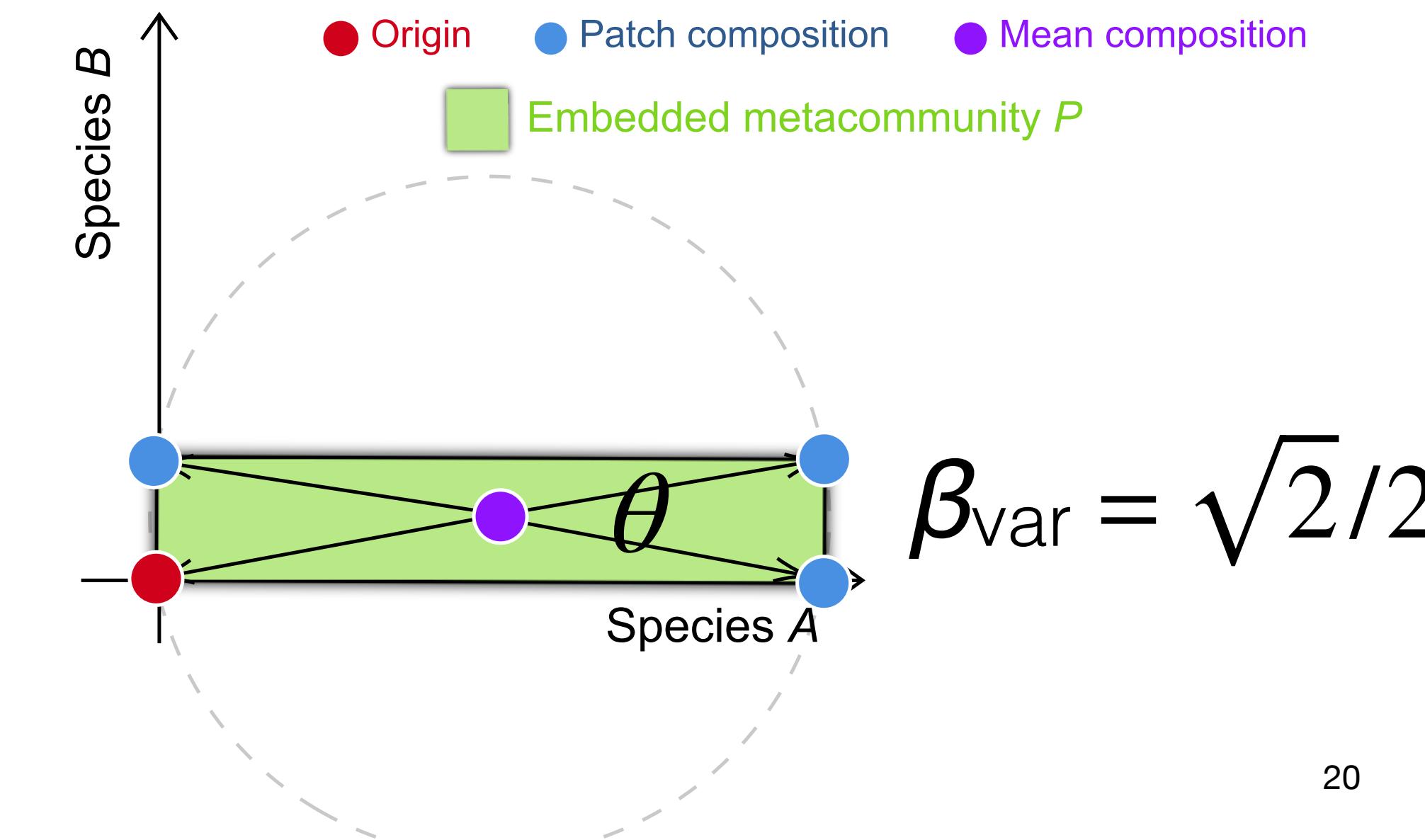
Links to classic measures



Linking β_{vol} to variance-based measure

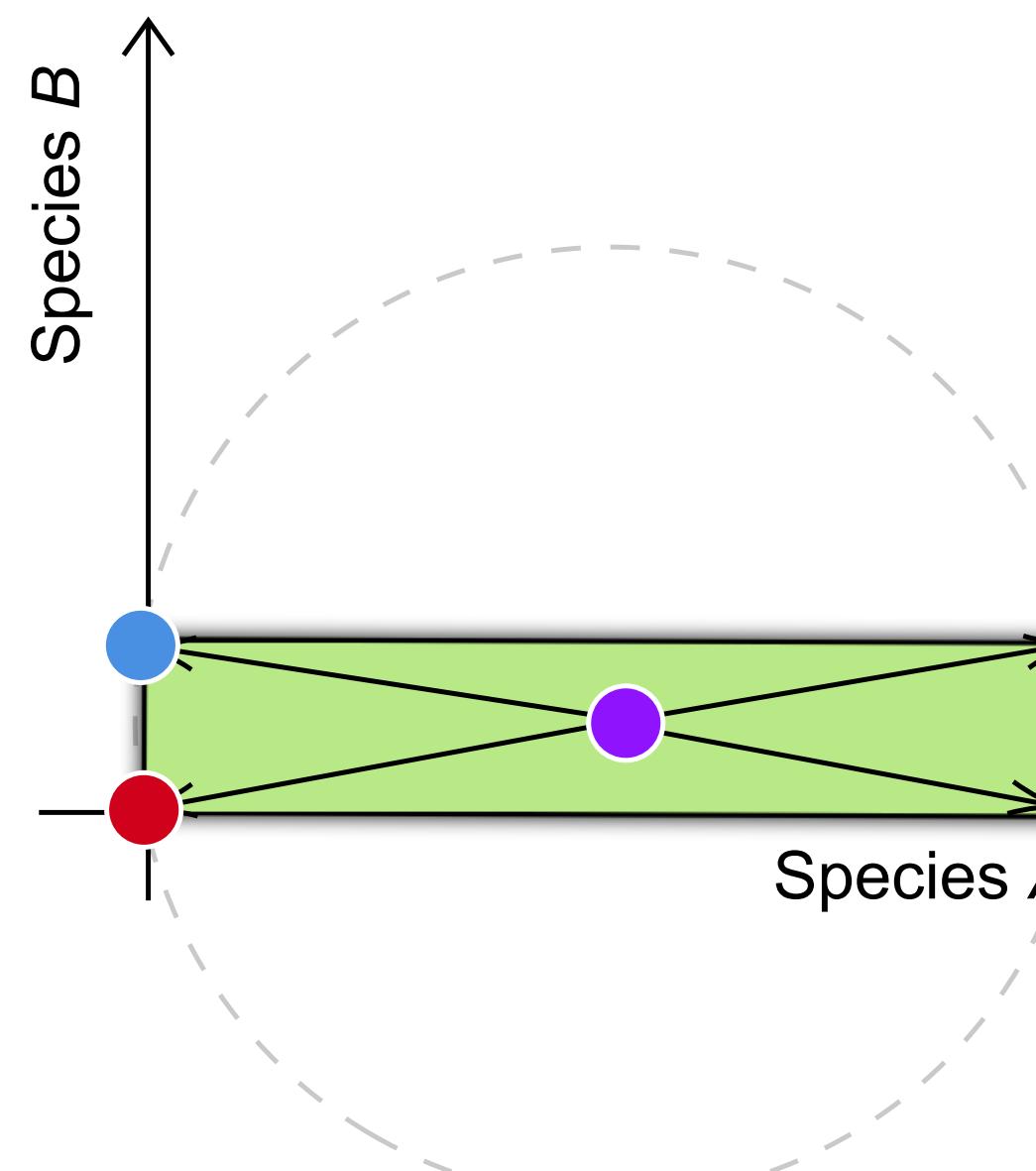
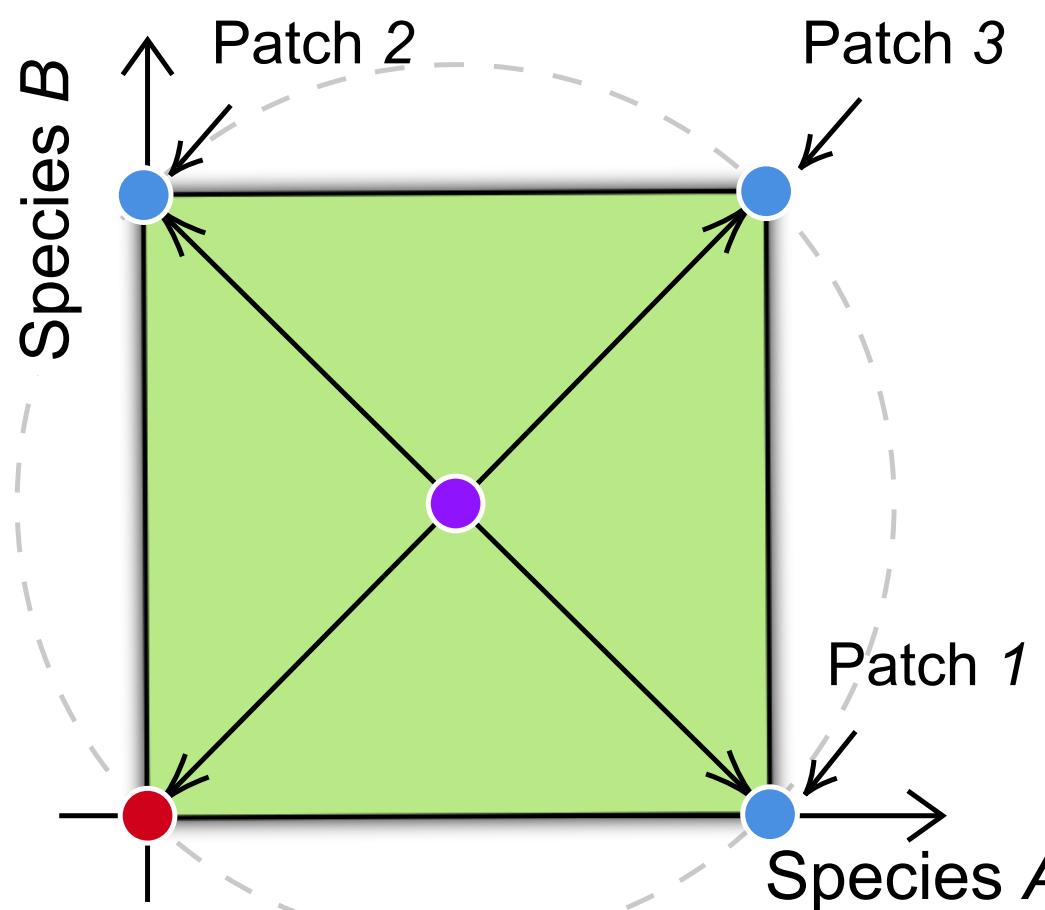


$$\beta_{\text{var}} = \sqrt{2}/2$$



$$\beta_{\text{var}} = \sqrt{2}/2$$

Linking β_{vol} to variance-based measure



Variance-based

First order information

Length

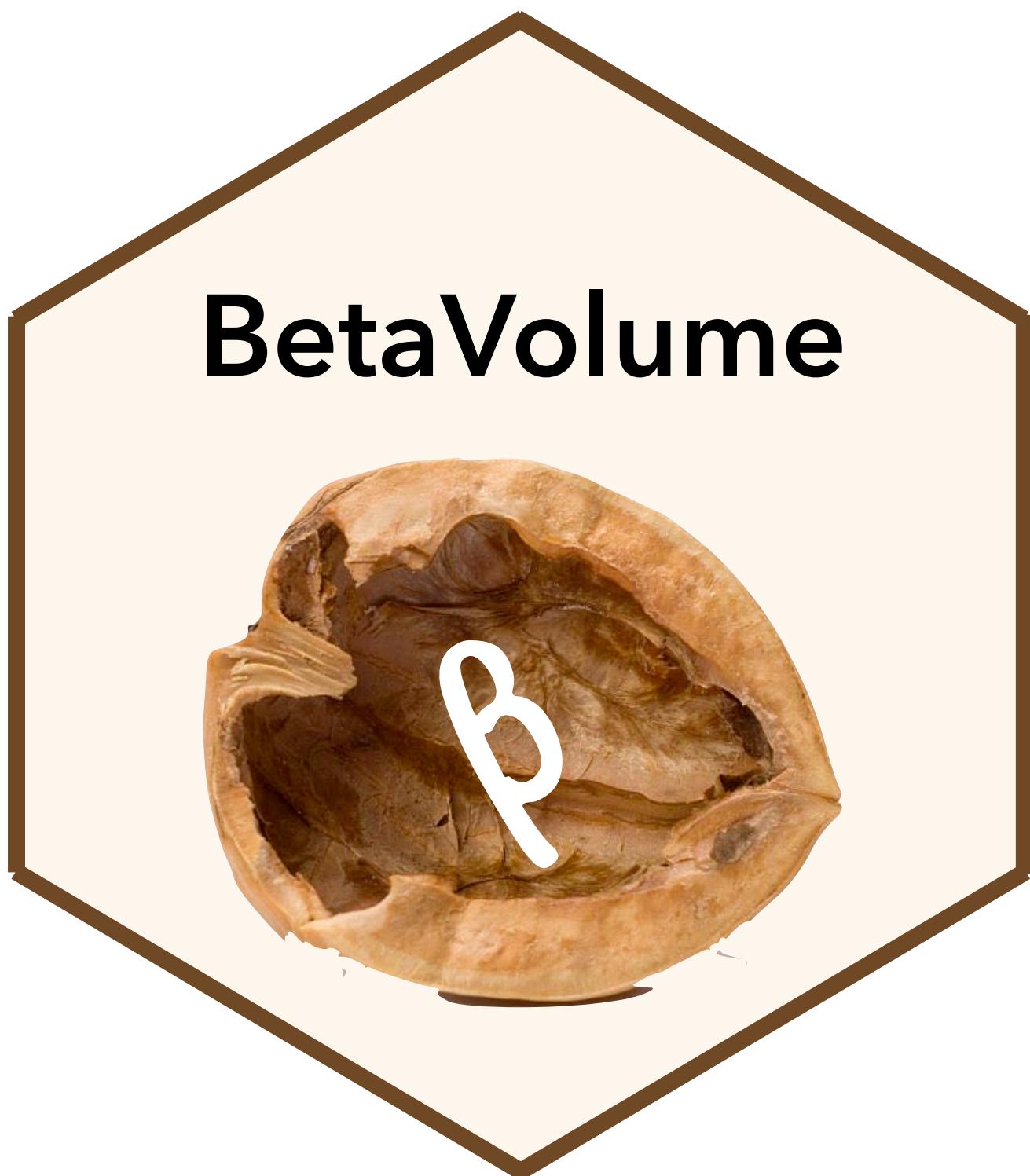
Second order information

Spatial association
 θ

β_{vol}

Empirical application

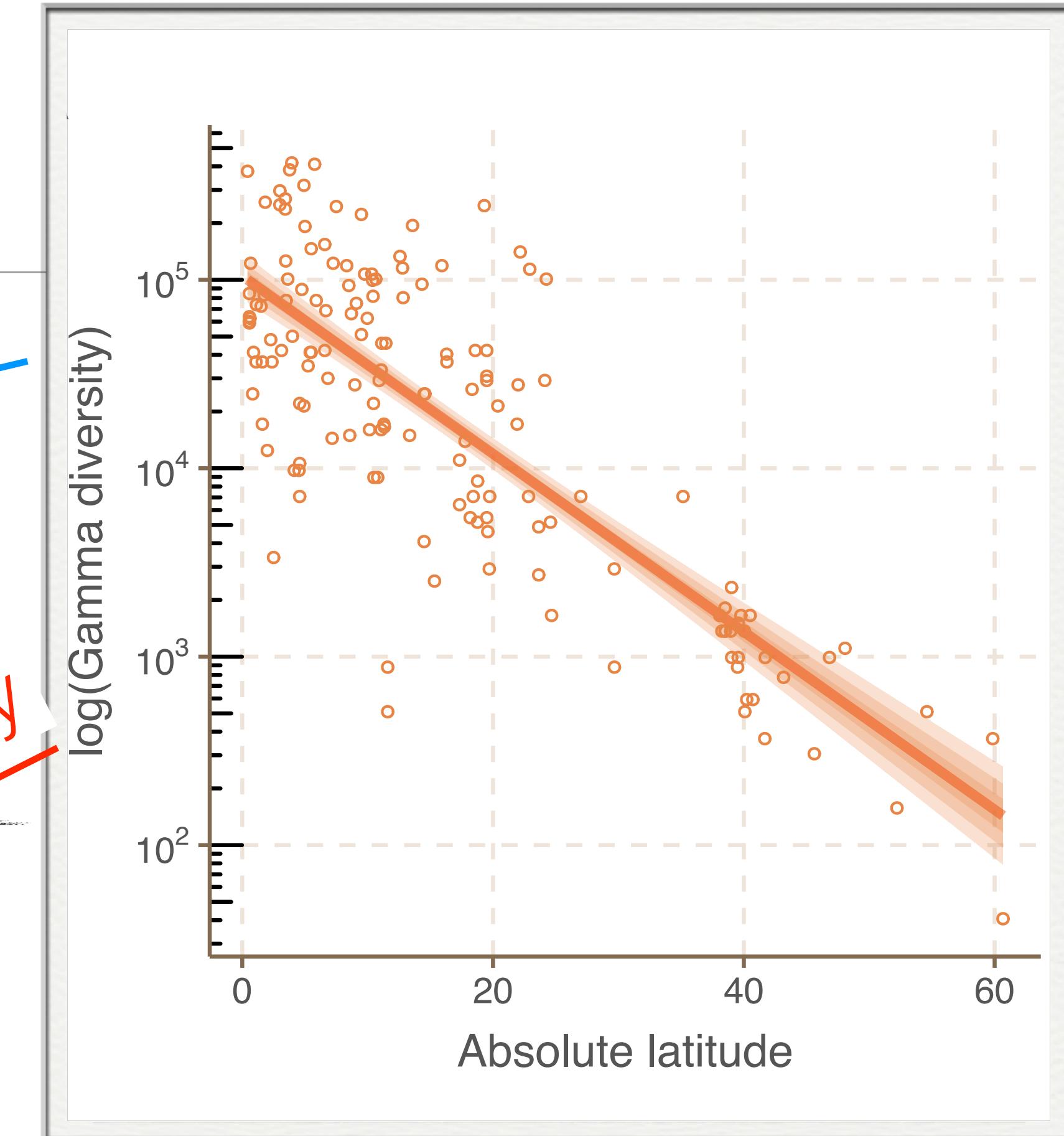
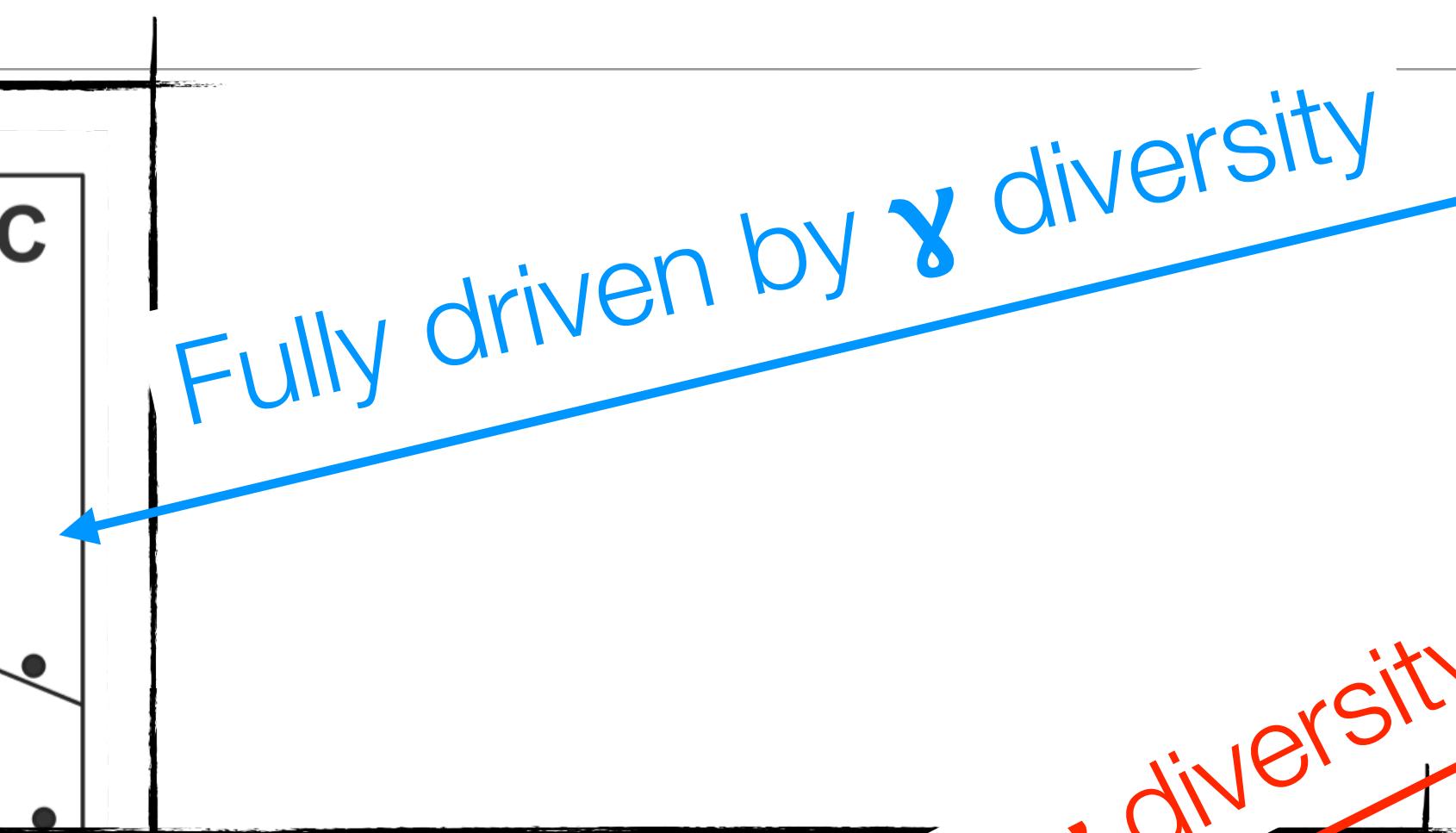
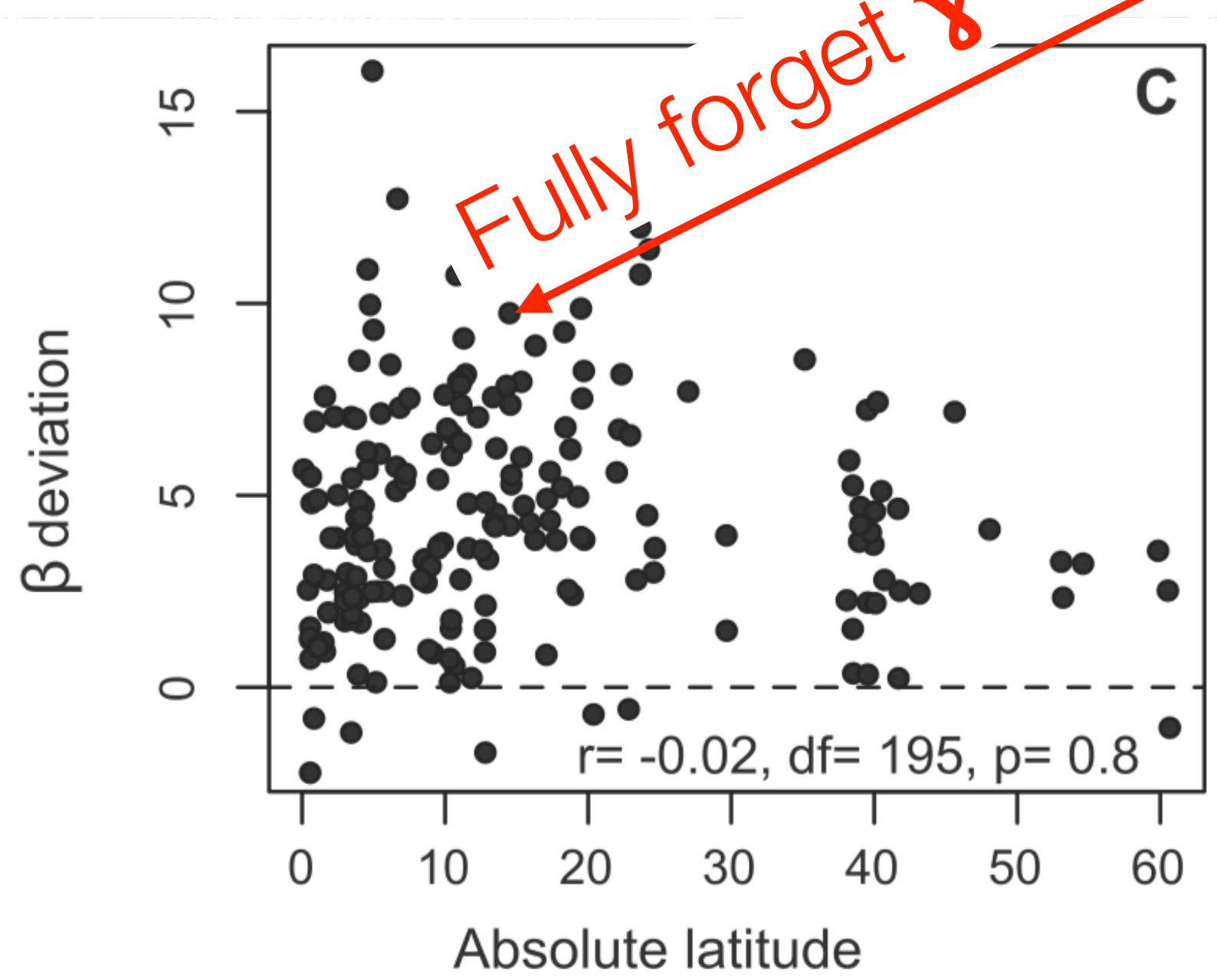
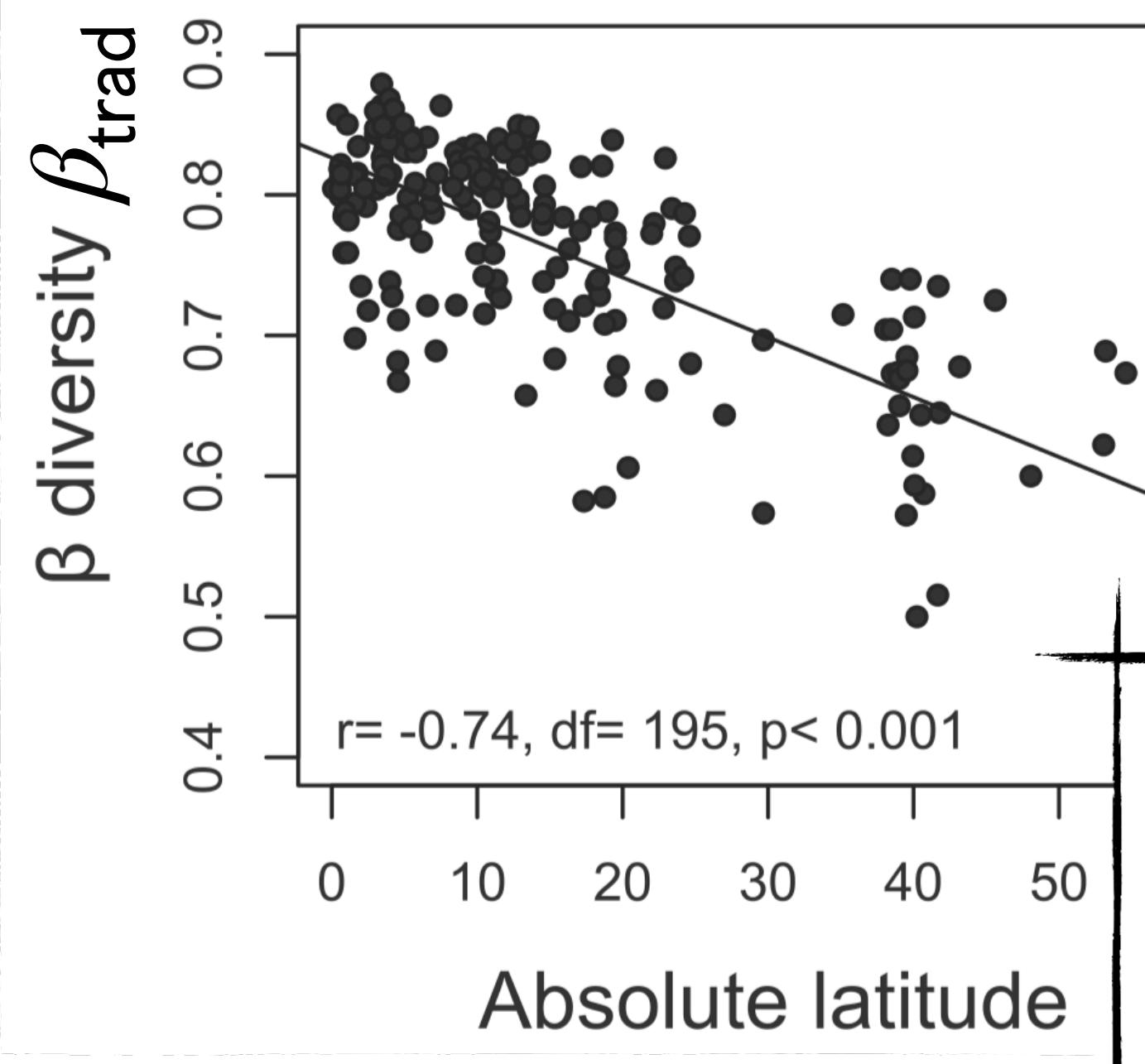
BetaVolume: An R package to compute β_{vol} efficiently



```
> library(betavolume)
> metacommunity
      [,1] [,2] [,3] [,4] [,5]
[1,]    1    0    0    1    0
[2,]    0    1    0    0    0
[3,]    0    1    1    1    1
[4,]    1    1    0    0    0
> beta_volume(as.data.frame(metacommunity), weight = F)
[1] 1.807204
```

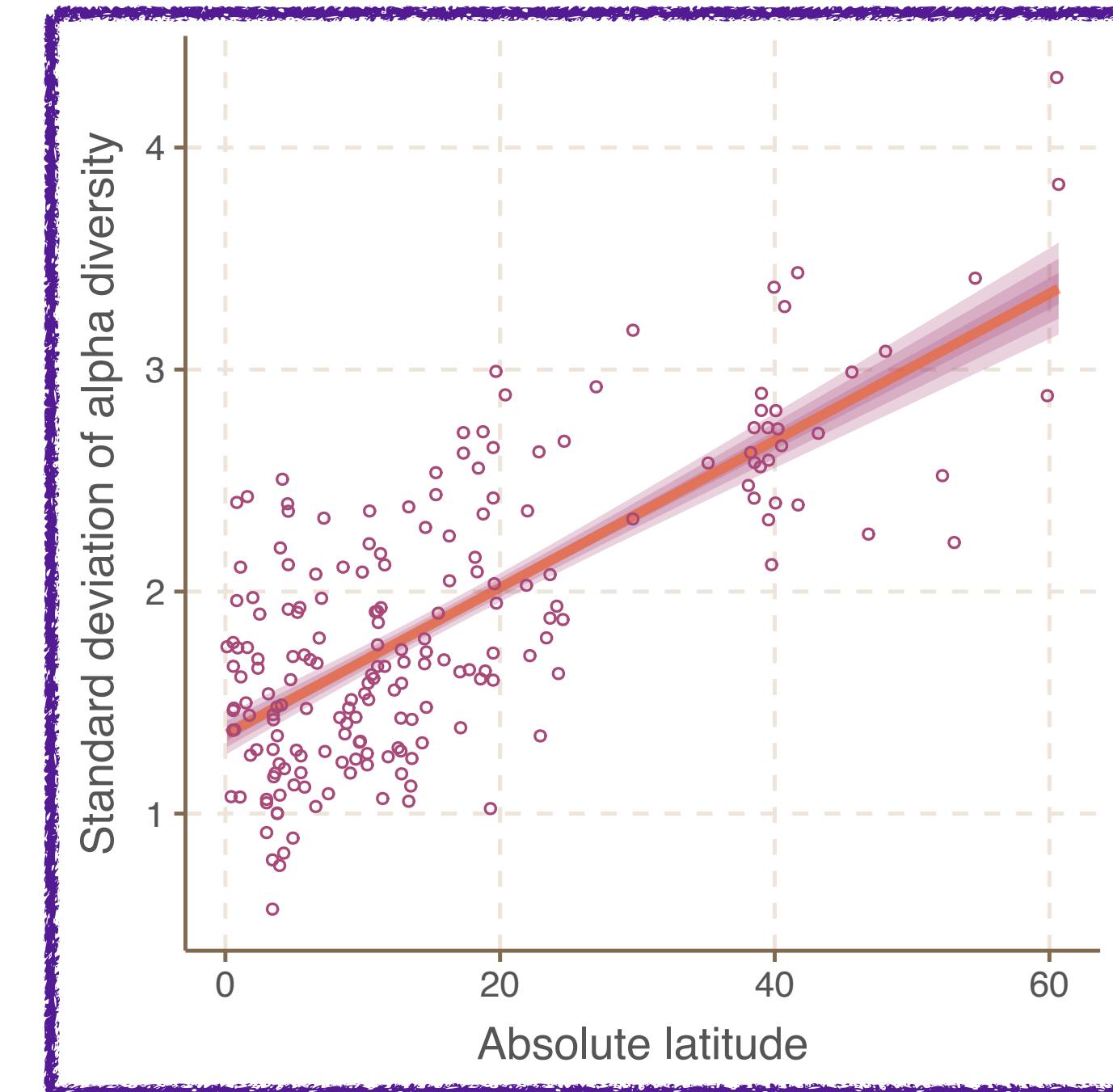
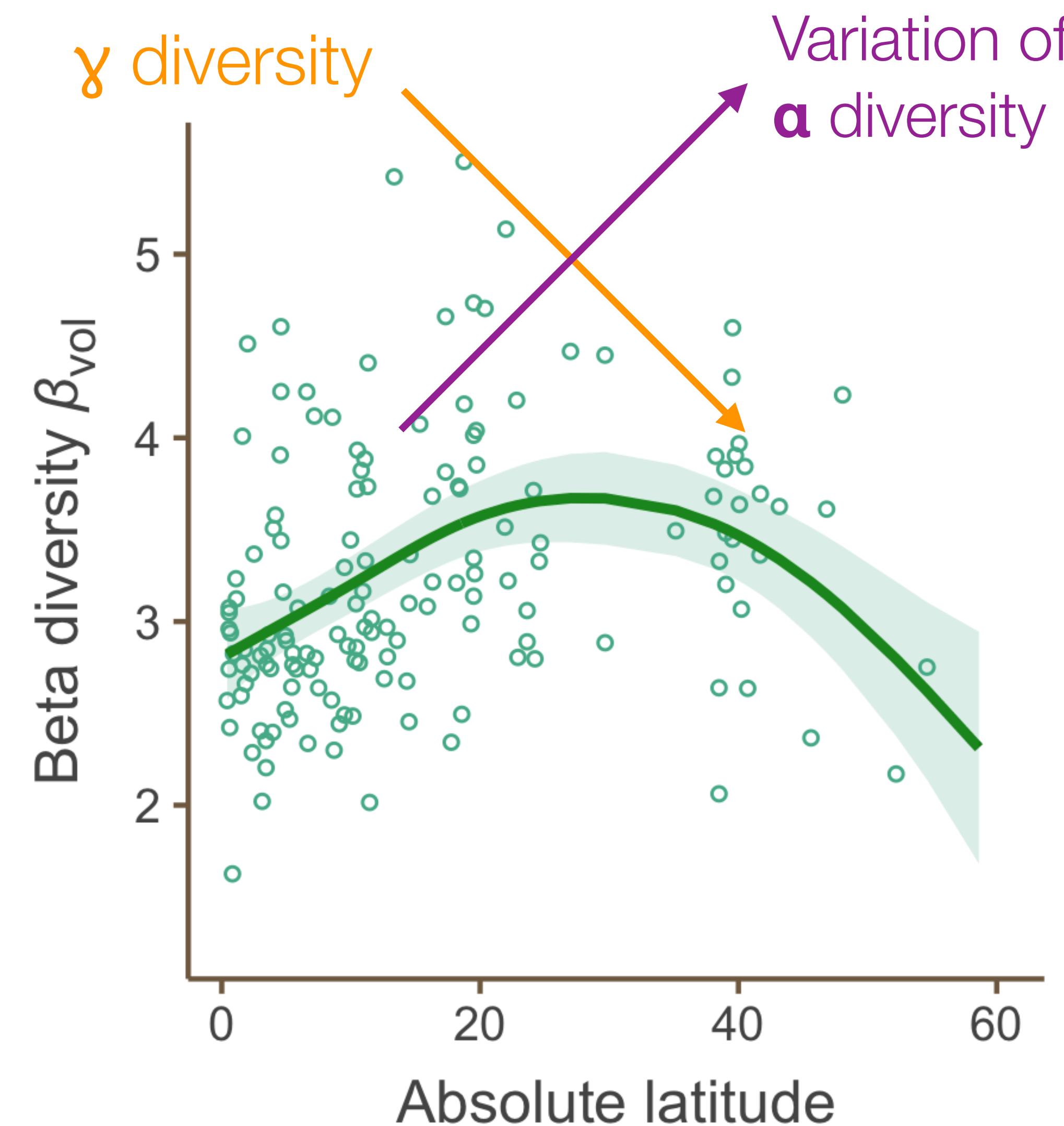
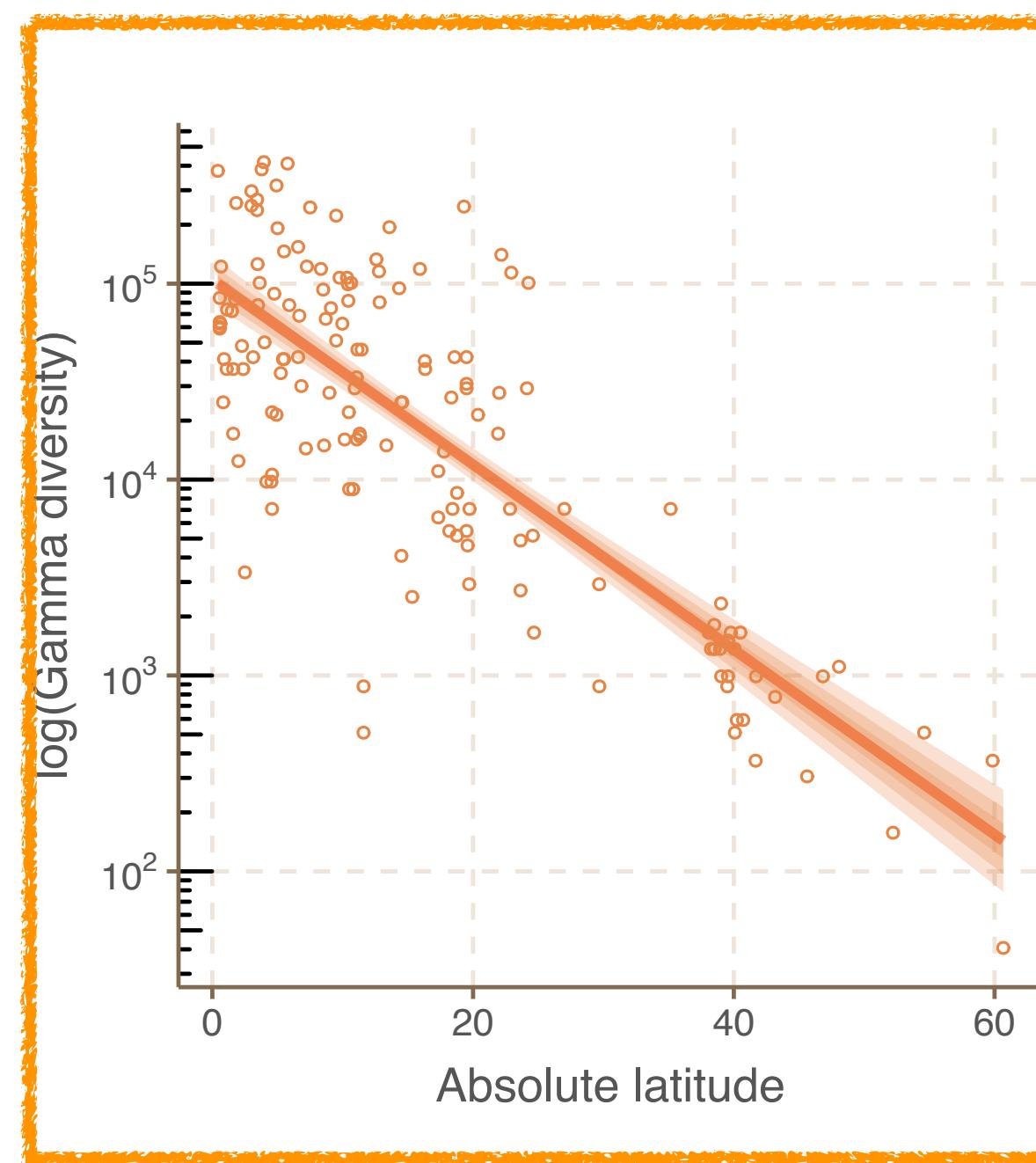
Beta diversity along latitudinal gradient

Nightmare with γ diversity



Beta diversity along latitudinal gradient

β_{vol} reveals a unimodal pattern



Sampling effects of beta diversity

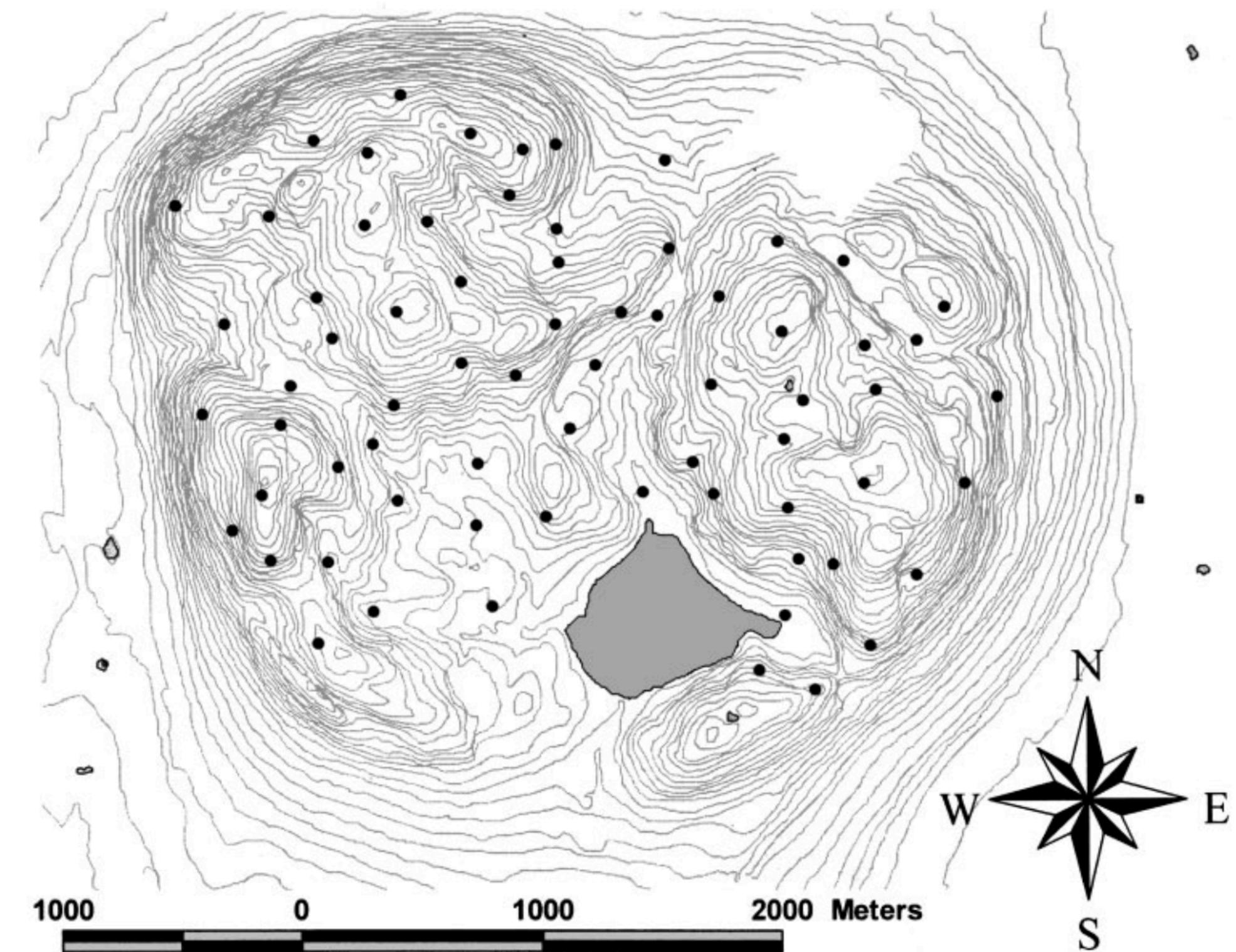
Sampling sites

Koffler Scientific Reserve



More homogeneous

Mont St. Hilarie

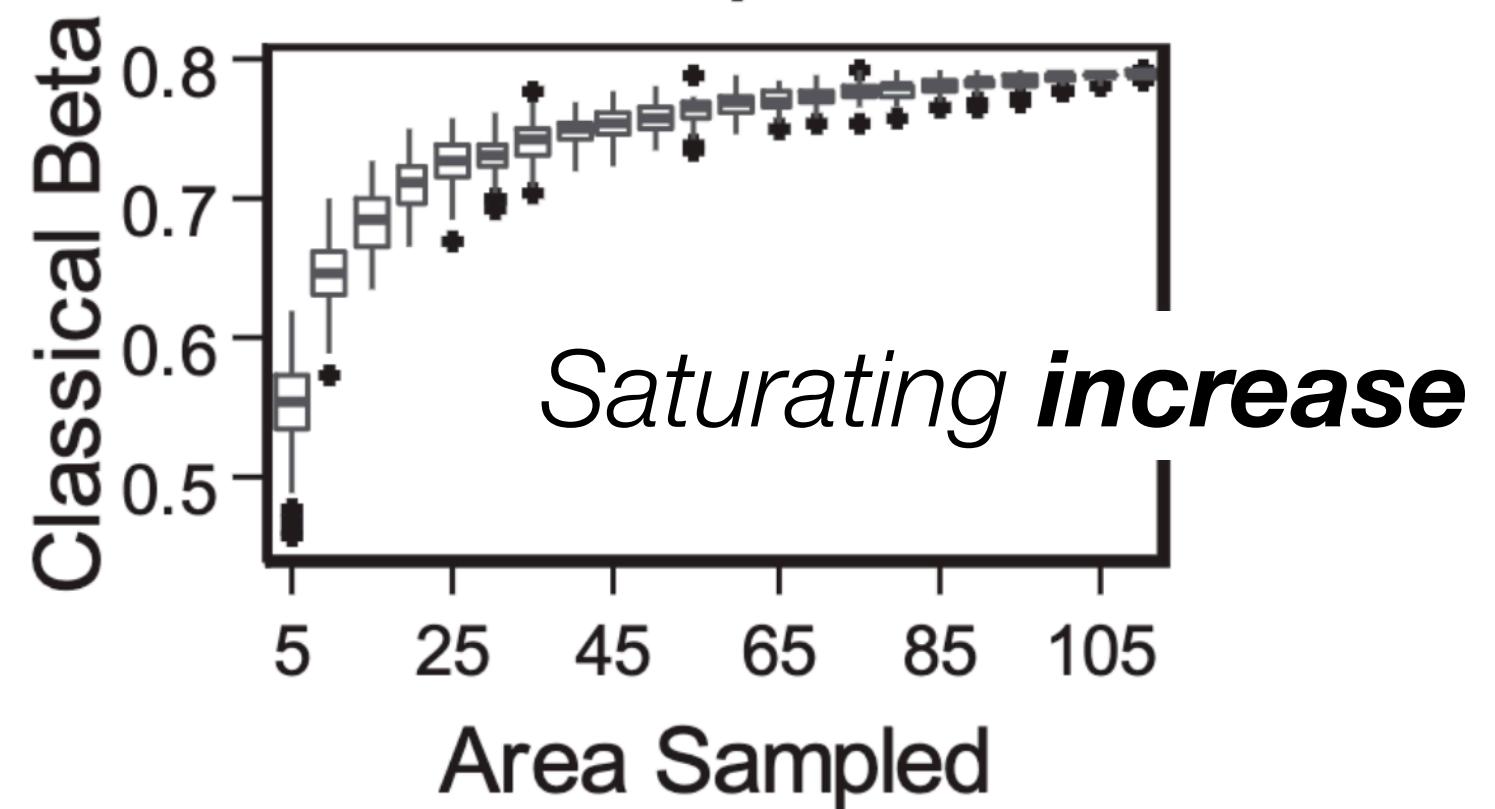


More heterogeneous

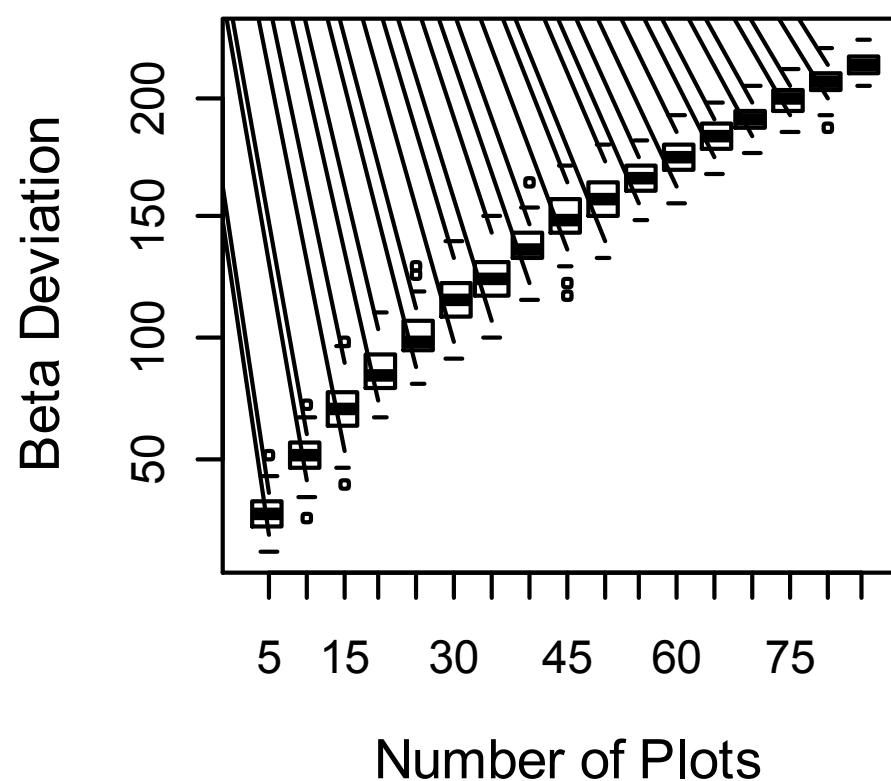
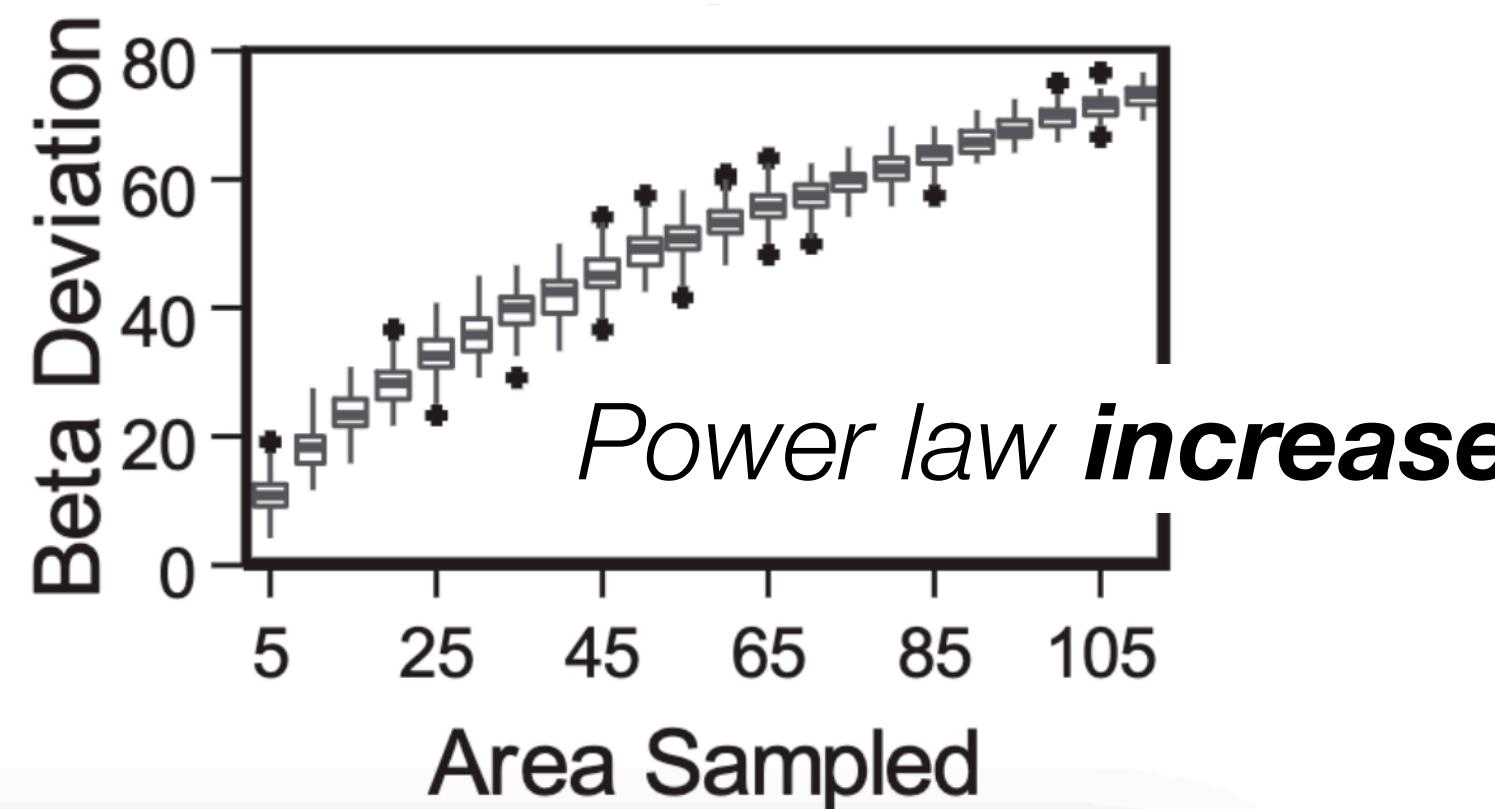
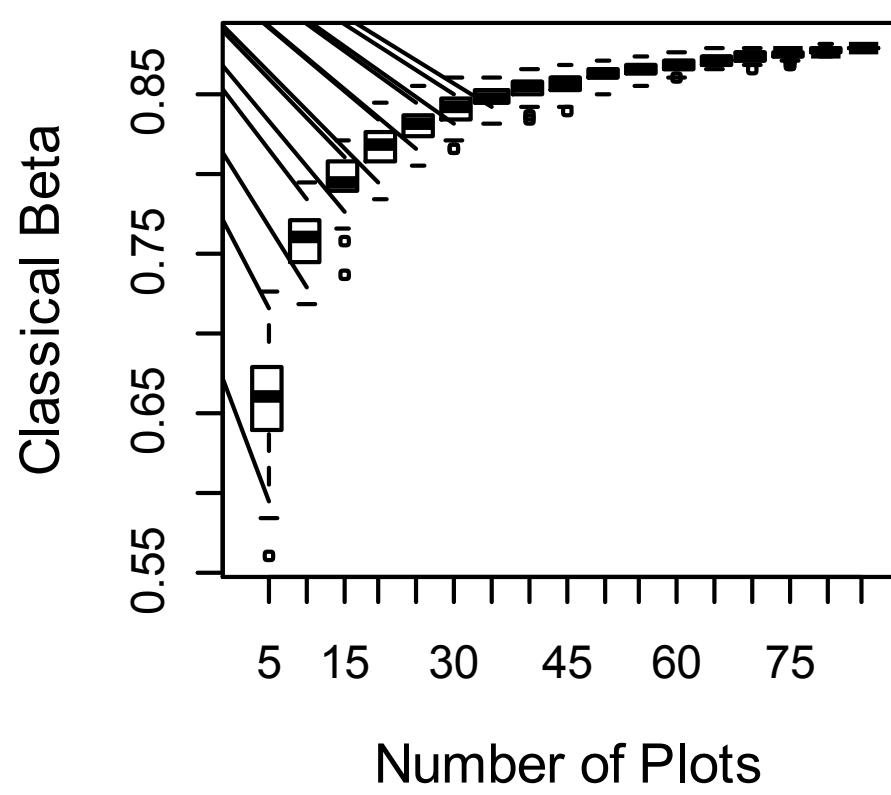
Sampling effects of beta diversity

Traditional beta diversity *always increases with sampling efforts*

Koffler Scientific Reserve



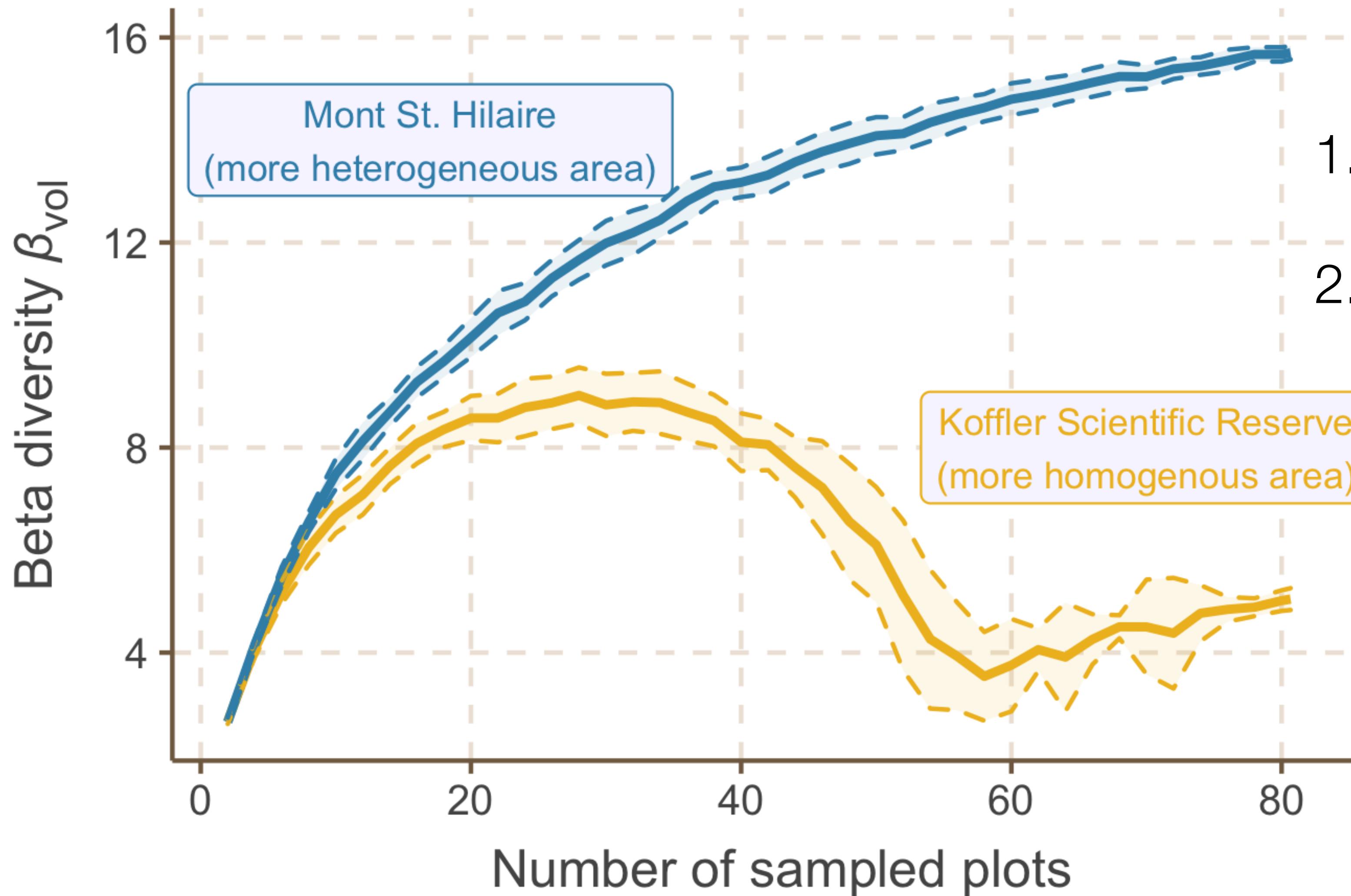
Mont St. Hilarie



1. More sampling does not provide more information
2. Fail to distinguish the levels of spatial variations

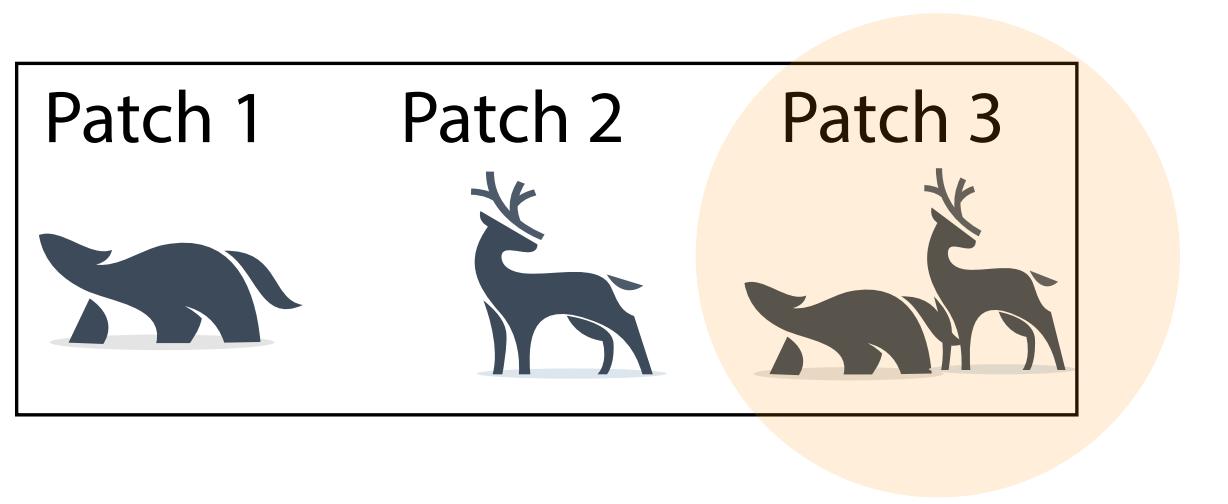
Sampling effects of beta diversity

β_{vol} reveals the spatial differences between the two sites



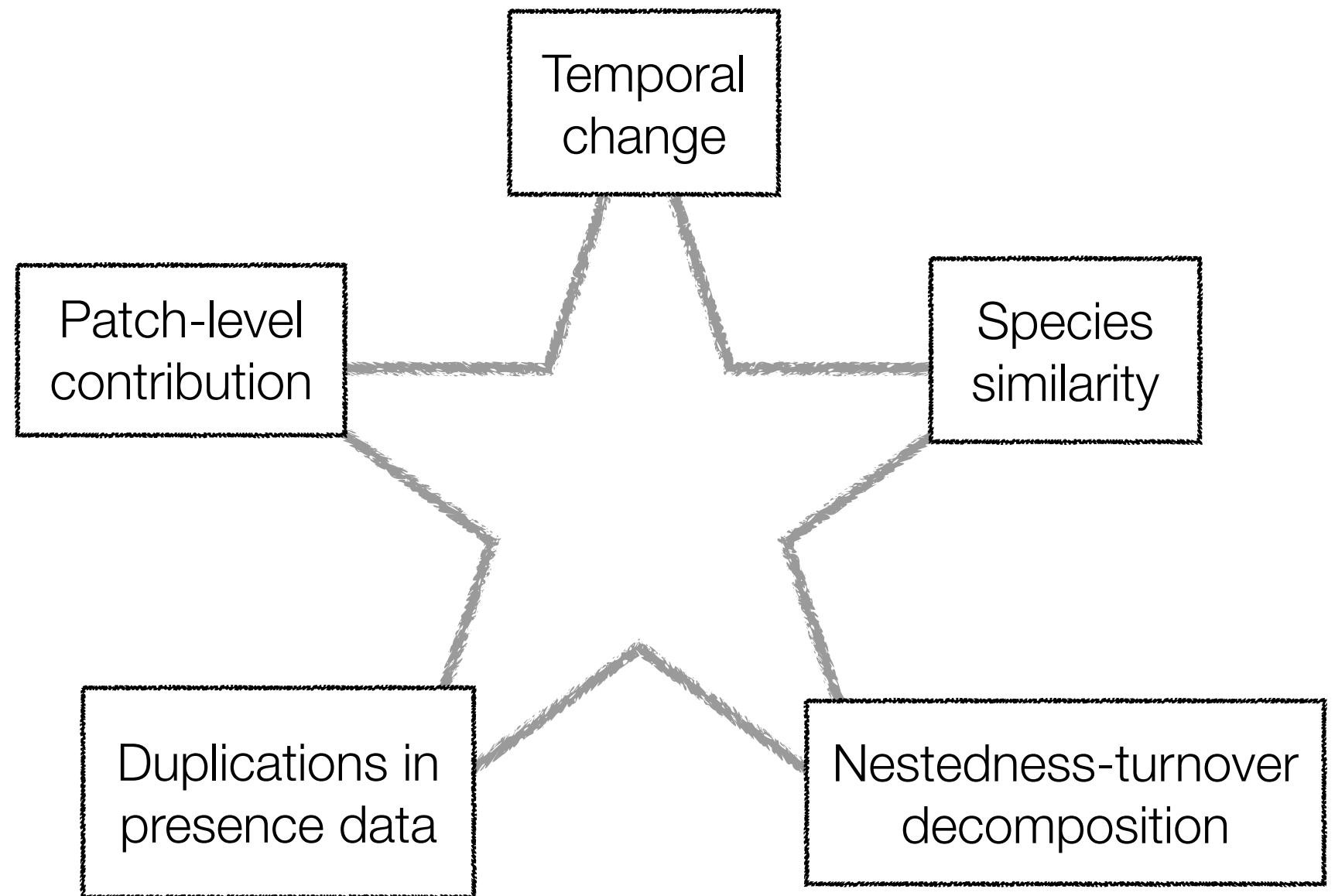
1. More sampling does ~~not~~ provide more information
2. ~~It's~~ distinguishes the levels of spatial variations





1. Traditional approaches ignored the non-additivity of ecological communities

3. A unified framework of beta diversity in various applications



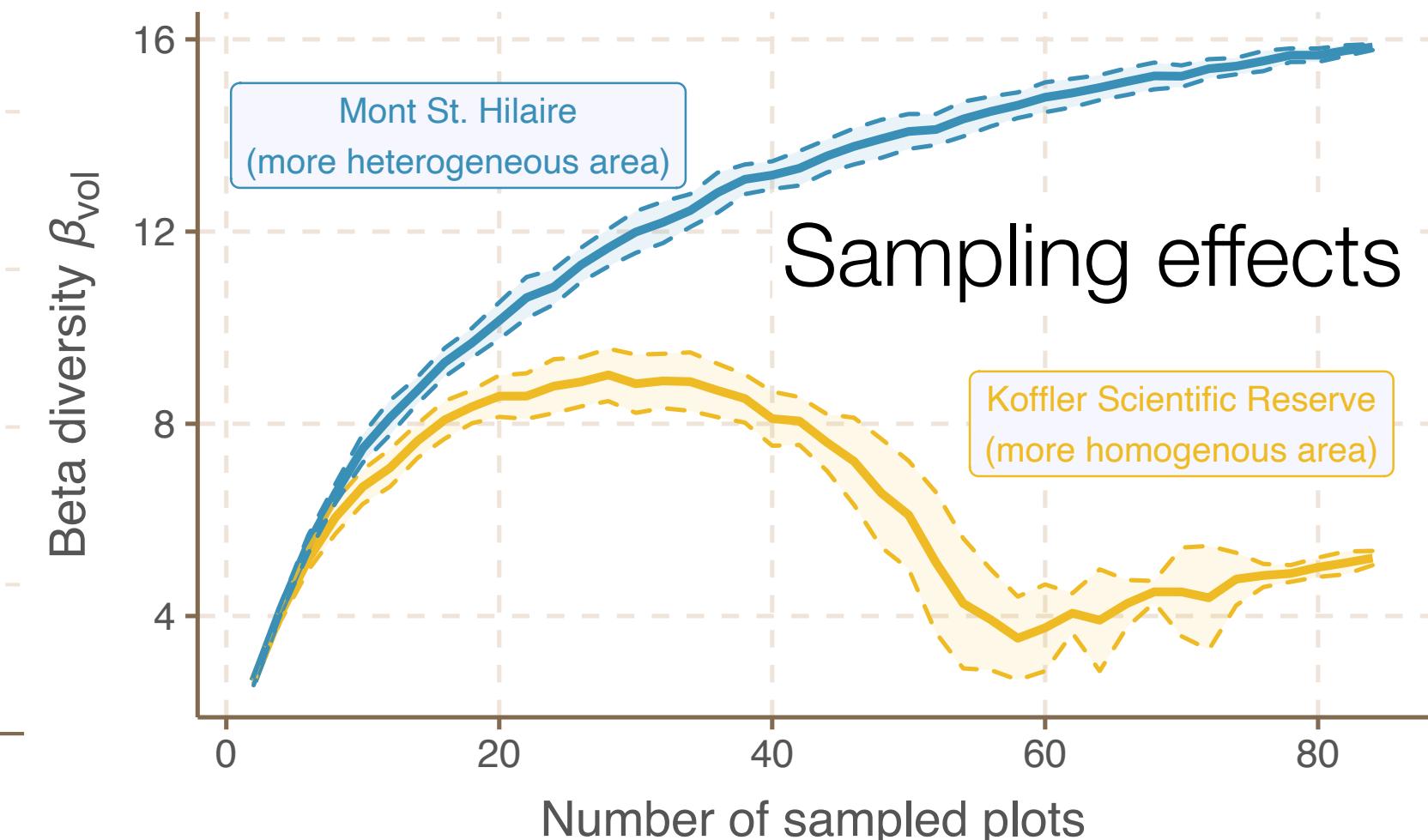
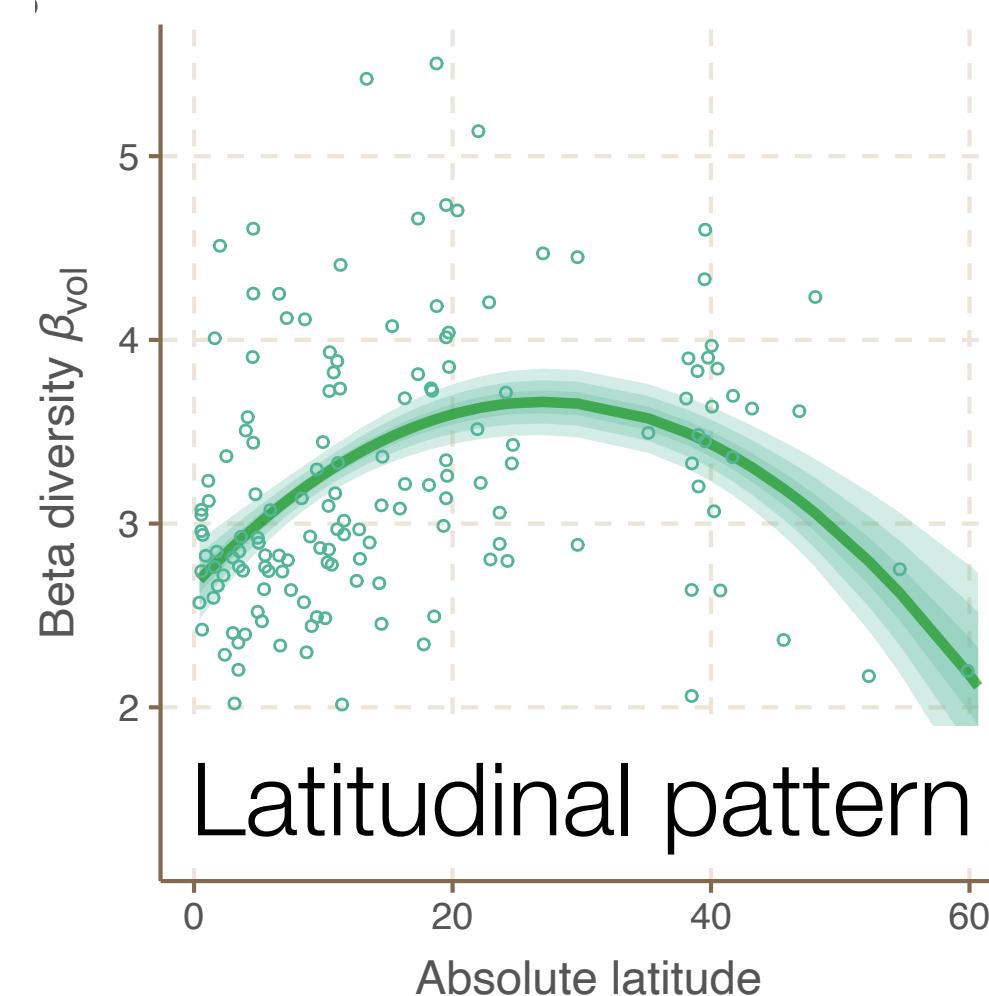
Embedded dimension

$$\beta_{\text{vol}} := d \times (\text{vol}(P))^{1/d}$$

Hypervolume

2. A geometric approach to measure beta diversity using hypervolume.

4. Provides novel insights to classic problems with beta diversity



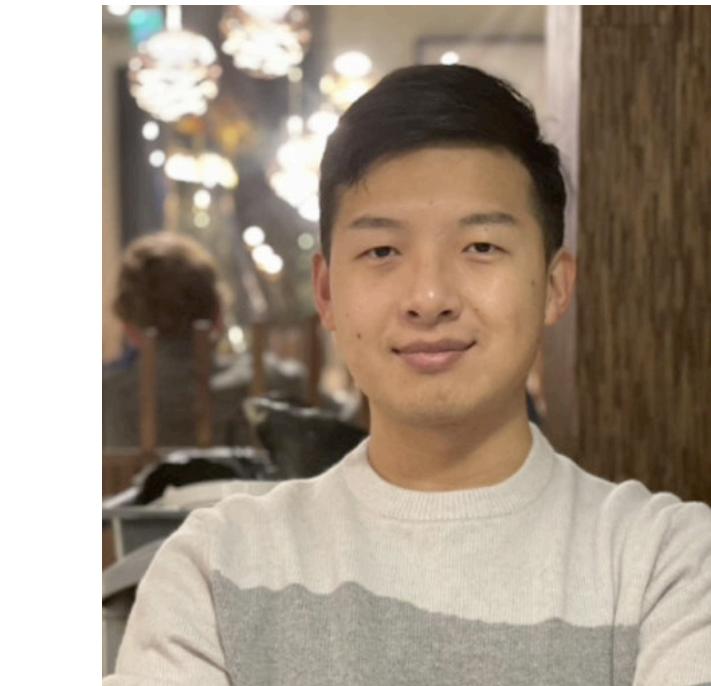
Beta diversity in a nutshell

a geometric approach to measure
community composition variations



Thank you!

Muyang Lu



Joseph Bennett



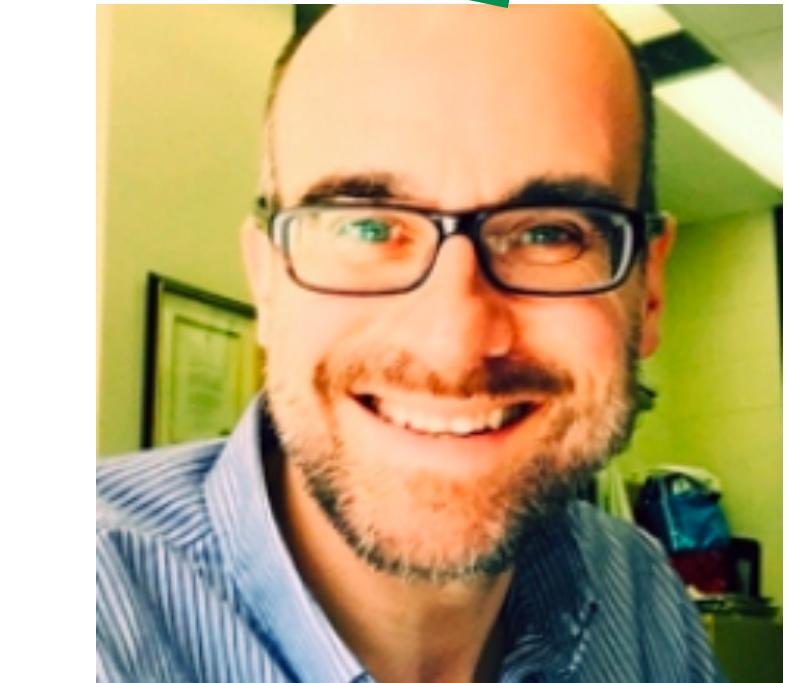
Benjamin Gilbert



Marie-Josée Fortin



Andrew Gonzalez



clsong.com



Chuliang_Song