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# ACCELERATING THE EMERGENCE OF ORDER IN SWARMING SYSTEMS

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Our ability to understand and control the emergence of order in swarming systems is a fundamental challenge in contemporary science. The standard Vicsek model (SVM) — a minimal model for swarming systems of self-propelled particles — describes a large population of agents reaching global alignment without the need of central control. Yet, the emergence of order in this model takes time and is not robust to noise. In many real-world scenarios, we need a decentralized protocol to guide a swarming system (e.g., unmanned vehicles or nanorobots) to reach an ordered state in a prompt and noise-robust manner. Here, we find that introducing a simple adaptive rule based on the heading differences of neighboring particles in the Vicsek model can effectively speed up their global alignment, mitigate the disturbance of noise to

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alignment, and maintain a robust alignment under predation. This simple adaptive model of swarming systems could offer new insights in understanding the prompt and flexible formation of animals and help us design better protocols to achieve fast and robust alignment for multi-agent systems.

*Keywords*: Swarming system; adaptive rule; Vicsek model.



## 1. Introduction

Collective motion describes the spontaneous emergence of ordered movement in a system of a large group of units or agents [45], such as insect swarms [30, 32], bird flocks [3, 28, 29], fish schools [19, 35], human crowds [14, 16], unmanned aerial vehicles (UAVs) [33, 46], and even migrating cells [40]. The most representative collective motion is *flocking* [37], which does not involve any central control [33]. For animals, being a member of a flock provides various advantages, e.g., minimizing vulnerability from predators [2, 20, 23, 39, 41], increasing foraging opportunities [8], and reducing the energetic cost of locomotion [25]. The engineering applications of flocking include massive mobile sensing [22], formation control of multi-agent systems [7, 31], self-organization of UAVs [38], and using the nanorobots to deliver active payloads for the diagnosis and treatment of cancer [4, 13, 15].

The intriguing phenomena of flocking have attracted the attention of physicists, biologists and engineers for decades. In particular, Vicsek *et al.* proposed a simple kinetic model [44] for self-propelled particles to reach *alignment*, a key ingredient for flocking [37]. In the standard Vicsek model (SVM), $N$ particles move with a constant speed $v_0$ in a square zone of size $L \times L$ with periodic boundary conditions. Each particle updates its heading (or direction) at next step based on the average velocity of its current neighbors within its sensing radius and random noise. The SVM contains three basic free parameters for a given system size $N$: (i) the noise level or magnitude $\eta$; (ii) the density of particles $\rho = N/L^2$; and (iii) the constant speed $v_0$ [44]. If the noise level $\eta$ is low enough, all particles will eventually reach consensus about their headings, i.e., align with each other. With increasing $\eta$, the system will gradually deviate from the perfect alignment state. If $\eta$ is high enough, particles will move in a completely disordered fashion, and the alignment is totally lost. Hence, SVM displays a continuous kinetic phase transition with increasing noise level $\eta$ [1, 44, 45].

Since Vicsek's pioneering work, many variants of SVM have been developed [5, 6, 9–11, 17, 18, 24, 26, 27, 33, 36, 42, 47–49]. In particular, some variants proposed adaptive updating rules to speed up the alignment by considering, e.g., a weighted average of velocities based on the time-varying number of neighbors of each particle [17], or an adaptive speed of each particle based on the local degree of alignment [24, 49]. A quick alignment is indeed very important for animals or general multi-agent systems to avoid obstacles [43] and minimize vulnerability of predators [2, 20, 23, 39, 41], because they can promptly adjust their states according to the emergence occurred during the motion. However, numerical simulations indicate that previous adaptive rules [24, 49] only slightly speed up the alignment (Fig. 1). In some parameter regime, the speed up is almost negligible. A fundamental question naturally arises: Is there a robust adaptive strategy that can drastically speed up the global alignment in all parameter regime and yield a robust alignment under noisy environment? In this paper, we address this fundamental question by proposing a general adaptive rule.

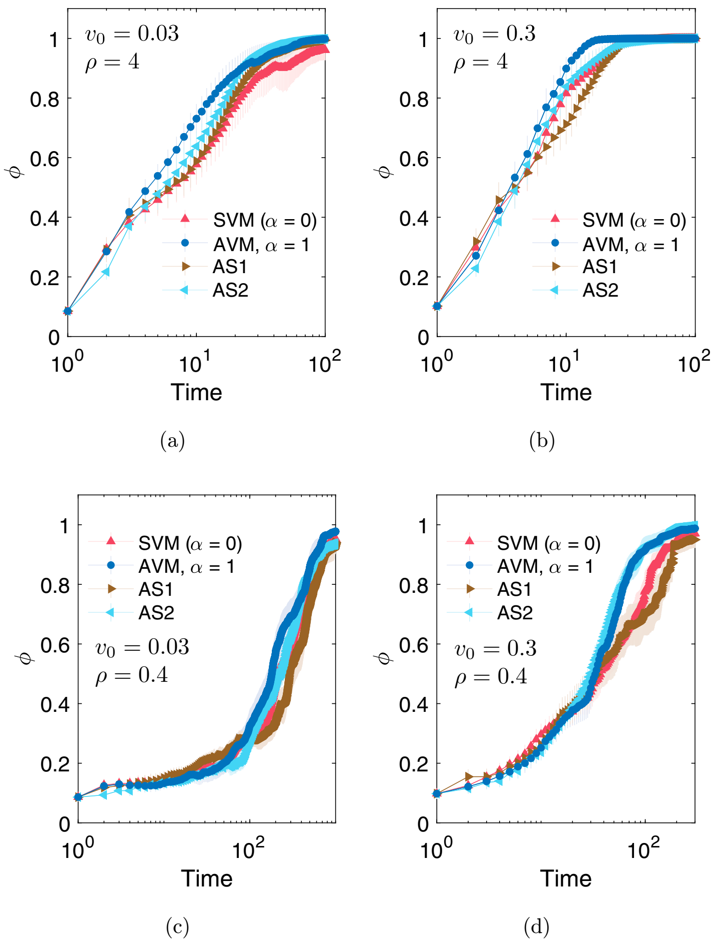



Fig. 1. **AVM can speed up the alignment in the noiseless case.** Order parameter $\phi$ as a function of time in SVM ($\alpha = 0$), AVM with $\alpha = 1$ and two other adaptive models proposed by [24, 49] (labeled as AS1 and AS2, respectively) in the absence of noise. The system parameters are $N = 100, \eta = 0$. AVM aligns faster than SVM, regardless of the different combinations of $\rho$ and $v_0$. (a) low speed, high density ($v_0 = 0.03, \rho = 4$). (b) High speed, high density ($v_0 = 0.3, \rho = 4$). (c) Low speed, low density ($v_0 = 0.03, \rho = 0.4$). (d) High speed, low density ($v_0 = 0.3, \rho = 0.4$). The data points are obtained by averaging over 10 different realizations. The error bar stands for standard deviation.

## 2. Standard Vicsek Model

In SVM, each particle moves towards a heading in a plane, which can be represented by a unit directional vector $e^{\iota\theta_i(t)}$. Here, $\theta_i(t)$ is the heading or angle of particle $i$ at step $t$, and the Greek letter $\iota$ represents the imaginary unit. Initially, all the particles are randomly distributed in a square zone of size $L \times L$ with headings following a uniform distribution in the interval $[0, 2\pi)$. The position of particle $i$ is then updated as

$$\mathbf{x}_i(t+1) = \mathbf{x}_i(t) + \mathbf{v}_i(t)\Delta t, \tag{1}$$

with periodic boundary conditions. The velocity of particle $i$ is given by $\mathbf{v}_i(t) = v_0 e^{\iota\theta_i(t)}$ and its angle $\theta_i(t)$ is updated as

$$\theta_i(t+1) = \arg\left[e^{\iota\theta_i(t)} + \sum_{j\in\mathcal{S}_i} e^{\iota\theta_j(t)}\right] + \Delta\theta_i(t). \tag{2}$$

Here, 'arg' gives the angle between the positive real axis and the average heading vector; $\mathcal{S}_i$ represents all the neighboring particles (except $i$ itself) within a circle of sensing radius $r$ that is centered at particle $i$; $\Delta\theta_i(t)$ denotes the noise term randomly chosen from a uniform distribution in the interval $[-\eta\pi, \eta\pi]$ and $\eta \in [0, 1]$ represents noise level or magnitude. In order to measure the global alignment of the system, one can define the order parameter $\phi$ as

$$\phi(t) = \frac{1}{N}\left|\sum_{i=1}^{N} e^{\iota\theta_i(t)}\right|. \tag{3}$$

A larger value of $\phi$ indicates a better alignment. When $\phi = 1$, all the particles are moving in the same direction, i.e., the system reaches a perfect global alignment.

## 3. Adaptive Vicsek Model

As shown in Eq. (2), the neighbors of particle $i$ contribute *equally* to the averaging process. Since each particle can sense its neighbors' headings, it can naturally adjust each neighbor's contribution in the averaging process simply based on the heading differences between itself and its neighbors. Inspired by the fact that the adaptation of coupling gains enhances the synchronization of coupled oscillators on a complex network [12, 50], one can introduce two kinds of adaptive rules:

$$\theta_i(t+1) = \arg\left[e^{\iota\theta_i(t)} + \sum_{j\in\mathcal{S}_i} e^{\iota w_{ij}(t)\theta_j(t)}\right] + \Delta\theta_i(t), \tag{4}$$

or

$$\theta_i(t+1) = \arg\left[e^{\iota\theta_i(t)} + \sum_{j\in\mathcal{S}_i} w_{ij}(t) e^{\iota\theta_j(t)}\right] + \Delta\theta_i(t). \tag{5}$$

The former makes the neighbors contribute a totally different heading vector to the focal particle, while the latter makes the neighbors only scale the lengths of their heading vectors to the focal particle.

For Eq. (4), suppose that $w_{ij}(t) = \Delta_{ij}(t)^{\alpha}$ for $j \in \mathcal{S}_i$, where $\Delta_{ij}(t) = \min\{|\theta_i(t) - \theta_j(t)|, 2\pi - |\theta_i(t) - \theta_j(t)|\}$ represents the absolute heading difference between particles $i$ and $j$, and $\alpha$ is the adaptation parameter. Note that (i) if $\alpha = 0$, then $w_{ij}(t) = 1$ for any $j \in \mathcal{S}_i$, the adaptation reduces to SVM; (ii) if $\alpha > 0$ and all the neighbors are in the same direction as the centered particle $i$ at time $t$, i.e., $\forall j \in \mathcal{S}_i$, $\theta_i(t) = \theta_j(t)$, then $w_{ij}(t) = 1$; (iii) if $\alpha < 0$ and $\theta_i(t) = \theta_j(t)$, then we set $w_{ij} = 1$ for $j \in \mathcal{S}_i$. Interestingly, this adaptive rule can introduce an effective external field to particles. For example, if $\theta_i \approx \theta_j$, $w_{ij}$ could be close to zero and $e^{\iota w_{ij}(t)\theta_j(t)} \approx 1$. That is, once particle $j$ follows almost the same heading of the focal particle $i$, the system could not be stable and is heavily attracted to the direction of $\theta \approx 0$. Our numerical simulations demonstrated that Eq. (4) can drastically accelerate the emergence of order in the swarming system. Yet, the price to pay is that all the particles are always forced to go in the same direction.

Hereafter we introduce the exact form of adaptive rule $w_{ij}$ in Eq. (5). We consider that

$$w_{ij}(t) = (\sin \Delta_{ij}(t))^{\alpha}, \tag{6}$$

for $j \in \mathcal{S}_i$, where $\Delta_{ij}(t) = \min\{|\theta_i(t) - \theta_j(t)|, 2\pi - |\theta_i(t) - \theta_j(t)|\} \in [0, \pi]$ and $\alpha \geq 0$. In this case, $w_{ij} \in [0, 1]$. If $\alpha = 0$, then $w_{ij}(t) = 1$ for any $j \in \mathcal{S}_i$, the adaptation reduces to SVM. For $\theta_i \approx \theta_j$, this adaptive rule avoids the introduction of an effective external field because $w_{ij}$ only scales the length of $e^{\iota\theta_j}$. Equation (6) indicates that

(i) if $0 < \Delta_{ij} < \pi/2$, with increasing $\Delta_{ij}$ the focal particle will adapt more contribution from neighbor particles.
(ii) if $\pi/2 < \Delta_{ij} < \pi$, with increasing $\Delta_{ij}$ the focal particle will decrease the contribution from neighbor particles.
(iii) $\Delta_{ij} = 0$ indicates no need of adaptive effect for the focal particle. $\Delta_{ij} = \pi$ means zero adaptive effect because the heading difference between particle $i$ and particle $j$ are too huge to be necessary to consider the adaptive effect from particle $j$. $\Delta_{ij} = \pi/2$ represents that the adaptive effect from particle $j$ reaches the maximum.

Following this adaptive rule, the focal particle adapts the contributions from neighbors based on the differences of their headings. At first, the contribution increases with the increment of heading differences between neighbors and focal particle. Once the difference is larger than a threshold ($\pi/2$), the contribution will decrease and eventually will become zero. Hereafter, we introduce the adaptive rule of Eqs. (5) and (6) to Vicsek model, and call it the adaptive Vicsek model (AVM).

## 4. AVM aligns much faster than SVM

To systematically check if AVM aligns faster than SVM and other variants in the absence of noise, we run simulations with different combinations of density $\rho$ and

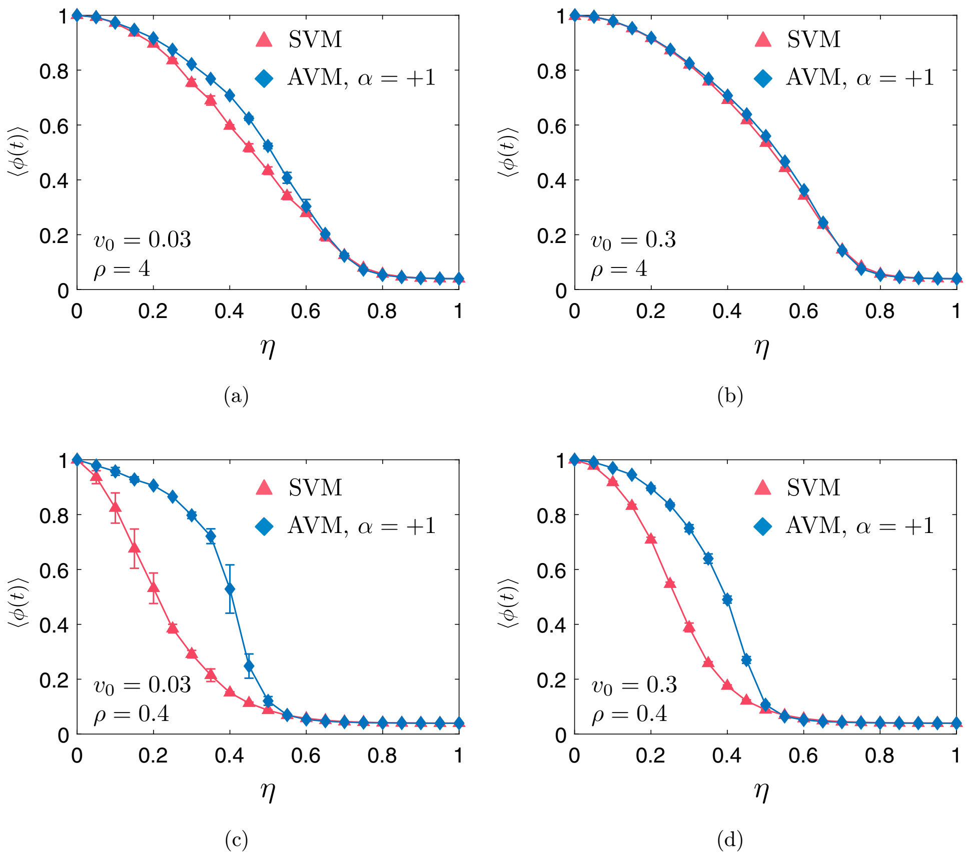



Fig. 2. **AVM can maintain the alignment of particles in the noisy environment.** The average of order parameter $\langle\phi(t)\rangle$ (average started from $t = 3 \times 10^4$ to $t = 5 \times 10^4$) as a function of noise level $\eta$. (a) low speed, high density ($v_0 = 0.03, \rho = 4$). (b) High speed, high density ($v_0 = 0.3, \rho = 4$). (c) Low speed, low density ($v_0 = 0.03, \rho = 0.4$). (d) High speed, low density ($v_0 = 0.3, \rho = 0.4$). The system parameters are $N = 500$, and time lasts $5 \times 10^4$. AVM is much more robust than SVM, especially in the regime of low density regardless of low or high speed. The data points are obtained by averaging over five different realizations. The error bar stands for standard deviation.

speed $v_0$. The sensing radius $r$ is set to be the unit distance, i.e., $r = 1$, and the length of the square zone is determined by $L = \sqrt{N/\rho}$. Figure 1 shows the order parameter $\phi$ as a function of time $t$ under different regimes with low or high density and speed. We find that AVM aligns faster than SVM, regardless of the different combinations of $\rho$ and $v_0$.

The results shown in Fig. 1 indicate that AVM aligns much faster than SVM in the noiseless case. Besides, we compare our model to two adaptive variants of Vicsek model based on the adaptive speed and local polarity (labeled as AS1 [24] and AS2 [49]), which find that AVM works better in all (speed and density) regimes. To check if this still holds in the presence of noise, we calculate the time-averaged order

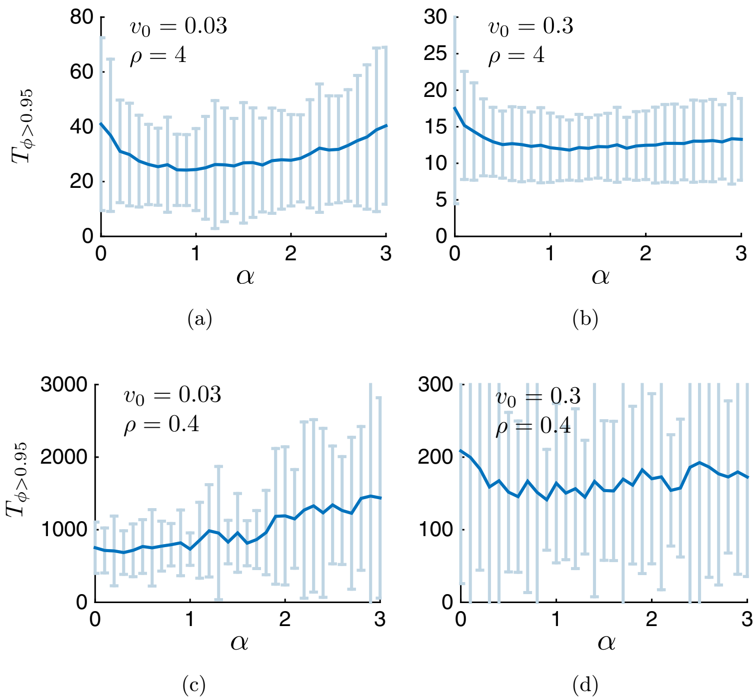



Fig. 3. **Adaptation parameters affect the alignment in the noiseless case.** $T_{\phi>0.95}$ as a function of $\alpha$. When $\alpha = 0$, AVM reduces to SVM. (a) Low speed, high density ($v_0 = 0.03, \rho = 4$). (b) High speed, high density ($v_0 = 0.3, \rho = 4$). (c) Low speed, low density ($v_0 = 0.03, \rho = 0.4$). (d) High speed, low density ($v_0 = 0.3, \rho = 0.4$). The system parameters are $N = 100$. The data points are obtained by averaging over 100 different realizations. The error bar stands for standard deviation.

parameter over the end of $2 \times 10^4$ steps, denoted as $\langle\phi(t)\rangle$, as a function of noise levels $\eta$ (see Fig. 2). We find that AVM is more robust than SVM in the presence of noise. Especially for the regime of low density (regardless of low or high speed, see Figs. 2(c) and 2(d)), AVM can maintain the alignment much better than SVM. Besides, from Eq. (6) we know that large $\alpha$ could not be suitable for AVM because $\sin(\Delta_{ij}(t)) \in [0, 1]$. Therefore, we systematically investigate how the adaptation parameter affects the emergence of order (see Fig. 3). The $y$-axis $T_{\phi>0.95}$ in Fig. 3 represents how long it will take the order parameter to be large than 0.95 when the particles start from a random configuration. In Fig. 3(a), $T_{\phi>0.95}$ first decreases and then increases with increasing $\alpha$. We find that $\alpha = 1$ is the best choice for all regimes.

## 5. Adaptive Rule Yields Robust Alignment Under Predation

To demonstrate the application of AVM in practical scenarios of swarming systems, we study the chasing-escaping process [2, 21, 34]. Consider predators invade the prey swarm to chase the prey and the prey can evade predators using an escape response

[2, 21]. It is crucial for the prey swarm to maintain the global order and have a robust flocking under predation. For simplicity, we assume that predator and prey have the same sense radius, i.e., $r = 1$, the same speed $v_0$; and the predation rules are:

- Predators: the predators only move along their initial headings during the whole process and do not interact with other predators and preys.
- Preys: if there were no predators in prey's sensing radius, the prey will move according to SVM or AVM. Otherwise, the prey will adopt the following escaping rules: (i) if the prey locates on the left (or right) side of the predator's heading $\theta_p$, the prey will adjust its heading as $\theta_p + \pi/2$ (or $\theta_p - \pi/2$), respectively, in the next step; (ii) if the prey is heading exactly towards the predator (which is very unlikely), the prey will adjust its heading as either $\theta_p + \pi/2$ or $\theta_p - \pi/2$ with equal probability in the next step; (iii) if the prey senses multiple predators, it will escape from the nearest predator. Note that once a prey detects predators in the sensing radius ($r = 1$), it will adopt the escaping rule at this step.

Of course one can design more complicated chasing and escaping rules. Here, we just use the most straightforward rule avoiding predators to demonstrate the impact of predation on the global order of the prey swarm.

Figure 4 shows the order parameter $\phi$ of the prey swarm to illustrate the difference between SVM and AVM under predation. Figure 4(a) shows that starting from an ordered state ($\phi \approx 1$ at the beginning), the order parameter of AVM and SVM both decrease but AVM is much slower than SVM, indicating that AVM can still mitigate

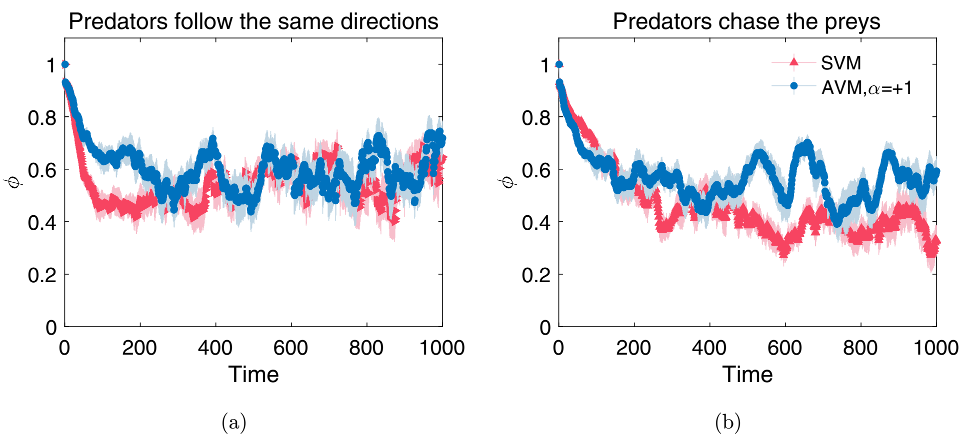
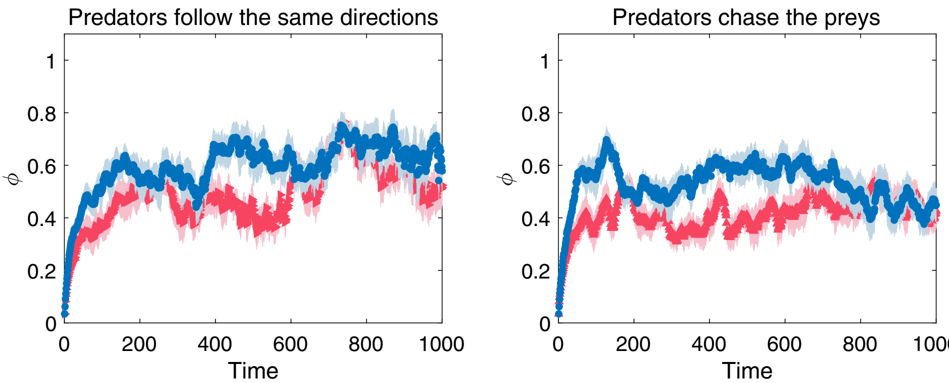



Fig. 4. **AVM maintains robust alignment under predation.** Order parameter $\phi$ of the prey swarm as a function of time for SVM and AVM with $\alpha = 1, N = 1200, \rho = 3, v_0 = 0.2, \eta = 0$, and $L = 20$. The number of predators is $N_\mathrm{p} = 10$. (a and c) The predators only move along their initial headings during the whole process and do not interact with each other. (b and d) The predator can adjust its heading in chasing its prey. (a and b) We assume that at the beginning the prey swarm is moving in an ordered state ($\phi \approx 1$). (c and d) The prey swarm is moving in a disordered state ($\phi \approx 0$) at the beginning. The data points are obtained by averaging over 10 different realizations. The error bar represents standard deviation.




(c)

(d)

Fig. 4. (*Continued*)

the negative effect on the prey swarm even under predation. Figure 4(b) shows that starting from a disordered state ($\phi \approx 0$ at the beginning), AVM can form the alignment much faster than SVM, and AVM can maintain the alignment of the prey swarm much better than SVM.

Next we consider a more complicated and realistic chasing rule that a predator can adjust its heading in chasing its nearest prey: $\mathbf{v}_p(t+1) = \mathbf{v}_s(t) - \mathbf{v}_p(t)$, where '$p$' represents predator, '$s$' represents prey, and $\mathbf{v}(t)$ represents the velocity vector at time step $t$. This means that the predator will take the direction directly pointing to the prey at the next step. If the predator senses multiple preys, it will chase the nearest predator. We assume the prey will adopt the same escaping rule as described above. We find that under this scenario AVM is still more robust under predation than SVM (see Figs. 4(c) and 4(d)).

## 6. Discussion

Our AVM displays a remarkable performance in accelerating the emergence of order, mitigating the effect of noise perturbation, maintaining the robust alignment under predation. For robots or UAVs, the fast alignment can reduce the time to form flocking at the beginning or after formation adjustment. The noise-robust property is more important for the motion of large population of units because noise is ubiquitous and sensing information with noise has a negative effect on decision making for agents or animals. Note that we only consider the external noise in the AVM. Further studies should consider more complicate and robust scenarios, such as intrinsic noise and possible delays. Furthermore, in the noisy environment AVM can stabilize the flocking and prevent the ordered regime being totally damaged by persistent noise. Finally, AVM maintains robust alignment of the prey swarm even under predation, which has a clear implication in practical problems related to swarming. In a word, due to the ability of fast alignment, rapid rearrangement of flocking and strong robustness to noise and predation, AVM has wide implications for understanding flocking behavior of animals, helping us design better algorithms for formation control of UAVs and efficient delivery of nanorobots. However, this AVM is based on numerical simulations and its theoretical analysis such as order convergence will be further studied in the future. Besides, our model is the phenomenological approach to mimic the flocking of real animal formations which lacks of considerations of physical and biological limitations, such as, the vision of distance and sense of neighbors. When transitioning between scenarios, i.e., from particles to animals, our model should consider the reality instead of the perfect particles.

## Authors Contribution

Y.-Y.L. conceived and designed the project. All authors performed the research. Y.X. performed all the numerical simulations. Y.X. and C.S. performed analytical calculations. All authors analyzed the results. Y.X. and Y.-Y.L. wrote the manuscript. C.S. and L. T. edited the manuscript.

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