

Appendix

Will a small randomly-assembled community be feasible and stable?

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S1 Equilibria in classic Lotka-Volterra systems

In this section, we first discuss the mathematical properties of equilibria in classic Lotka-Volterra (LV) systems, and then we provide evidence that the conclusions reached in our manuscript are robust to different methods to compute these equilibria.

A LV system is called degenerate if the determinant of the interaction matrix A is zero. Of course, this would make no biological sense since the population abundances will blow up (Case, 2000; Stone, 2016). Therefore, in line with this biological implication, it has been proven that the construction of the interaction matrix would almost never generate a singular matrix (Tao et al., 2010; Wood et al., 2012). In fact, the LV system has at least one fixed non-trivial (non-zero) equilibrium if the system is non-degenerate. This is true because $\text{diag}(N) \cdot (AN + r) = 0 \iff AN + r = 0$ if the abundances of all species are required to be strictly positive. Note that the non-trivial equilibrium is called interior if all equilibrium abundances (N^*) are strictly positive.

In addition to the non-trivial equilibria, there is also another type of equilibria in LV systems, what is known as border equilibria, which corresponds to the case where at least one of the constituent species abundance goes to zero (but not all). Importantly, the positivity property of LV systems guarantees that all species abundances will remain non-negative if the system is also initiated with non-negative abundances (Takeuchi, 1996). Thus, the LV system might converge to a non-trivial equilibrium (all abundances are non-zero) or to a border equilibrium (all abundances are non-negative).

Importantly, the stability analysis of a LV system does not only depend on the interaction matrix A , but also on the equilibrium abundances (N^*). Even if the LV system is globally stable, it still requires that the equilibrium abundances are strictly positive (Goh, 1977). In general, there are two common methodologies to find these equilibria. The first one is done by solving directly the nonzero equilibrium, which simplifies theoretical analysis, but it is likely to give negative equilibrium abundances (violating biological constraints). The second method is done by simulating the dynamics using the ordinary differential equations, which can eliminate the problem of negative abundances, but introduces the problem of establishing a criteria for border equilibria, and in some cases results can be dependent on initial values (Coyte et al., 2015; Bashan et al., 2016). Unfortunately, to our knowledge, there is no obvious link between these two methods.

In our manuscript, all results are generated via the first method (i.e., solving algebraically the LV system). Here, we reproduce our results using the second method (i.e., using simulations). See Fig. A1 to Fig. A2. Note that the value of all parameters are the same as in the manuscript, the only difference relies on the method to calculate the equilibria. The initial abundances for simulations are generated from a uniform distribution in $[0, 1]$. A species is considered extinct when its abundance is less than 10^{-14} . In the special case when the given parameterization gives unbounded abundances, these were discarded. Recall that the simulation method is applied to get the border equilibrium if the interior equilibrium does not exist, therefore just for the cases where the species growth rates are selected inside the feasibility domain, species abundances were not recalculated.

We focused on local stability as it is one of the most studied properties, and it is central to the debate presented in our manuscript (Roberts, 1974; Goh, 1977), although we acknowledge that many definitions of stability have been proposed to measure different aspect related to the dynamics of ecological communities (May, 2007; Ives and Carpenter, 2007). Local stability only assures that the system would return to equilibrium after an infinitesimal pulse perturbation (Strogatz, 2014). Of course, global stability is a much stronger condition that allows the system to return to equilibrium after a pulse perturbation of any given magnitude (Logofet, 1993); however,

there is not known easy criteria as sufficient and necessary condition for global stability of LV systems (Takeuchi, 1996). Volterra-dissipative matrices is one criterion, but this would only allow us to study a small subset of systems (Logofet, 1993), especially as our aim is to focus on the emergence of small randomly-assembled systems.

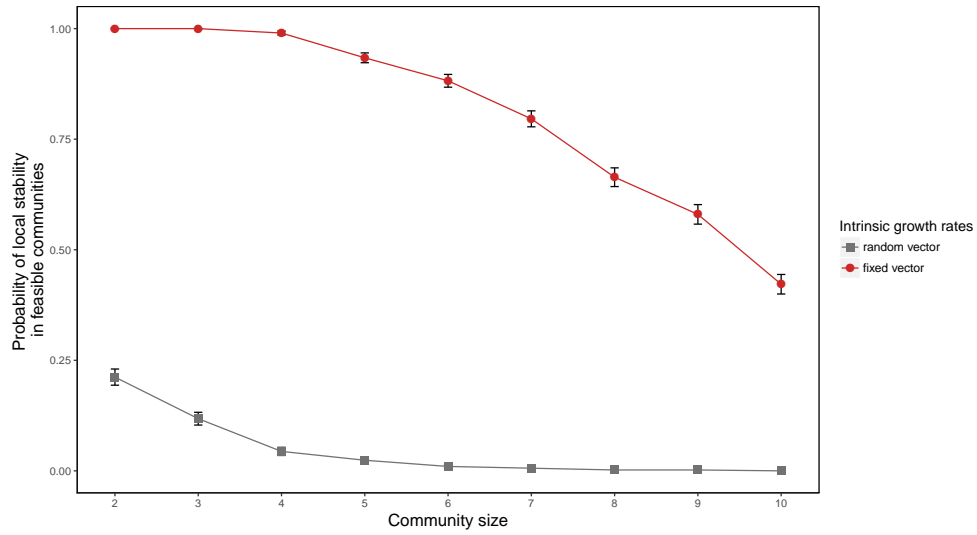


Figure S1: Equivalent to Figure 1a in the manuscript, but here the equilibrium is calculated via simulation.

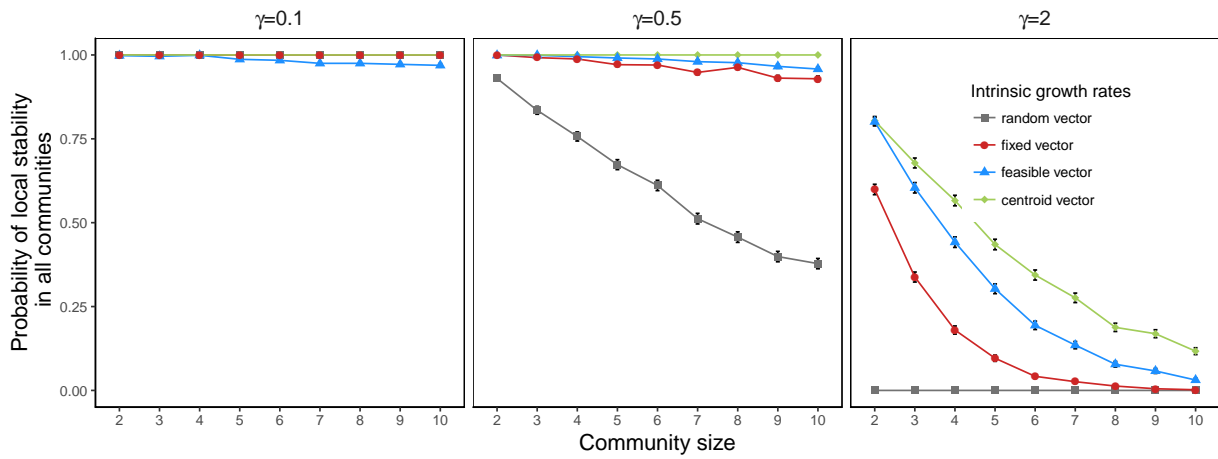


Figure S2: Equivalent to Figure 3 (Panels a-c) in the manuscript, but here the equilibrium is calculated via simulation.

S2 Dependency of previous results on sampling methods

As mentioned in the manuscript, previous results on the association between stability and feasibility are dependent on the sampling methods used to generate the interaction matrix A (Roberts, 1974; Gilpin, 1975; Goh and Jennings, 1977; Rozdilsky and Stone, 2001; Christianou and Kokkoris, 2008). Here we show this dependency. Specifically, Figures S3 to S6 reproduce the stability patterns for feasible systems changing the sampling distribution of interaction matrix A , interaction strength σ , and fraction of negative diagonal elements in the interaction matrix A , respectively.

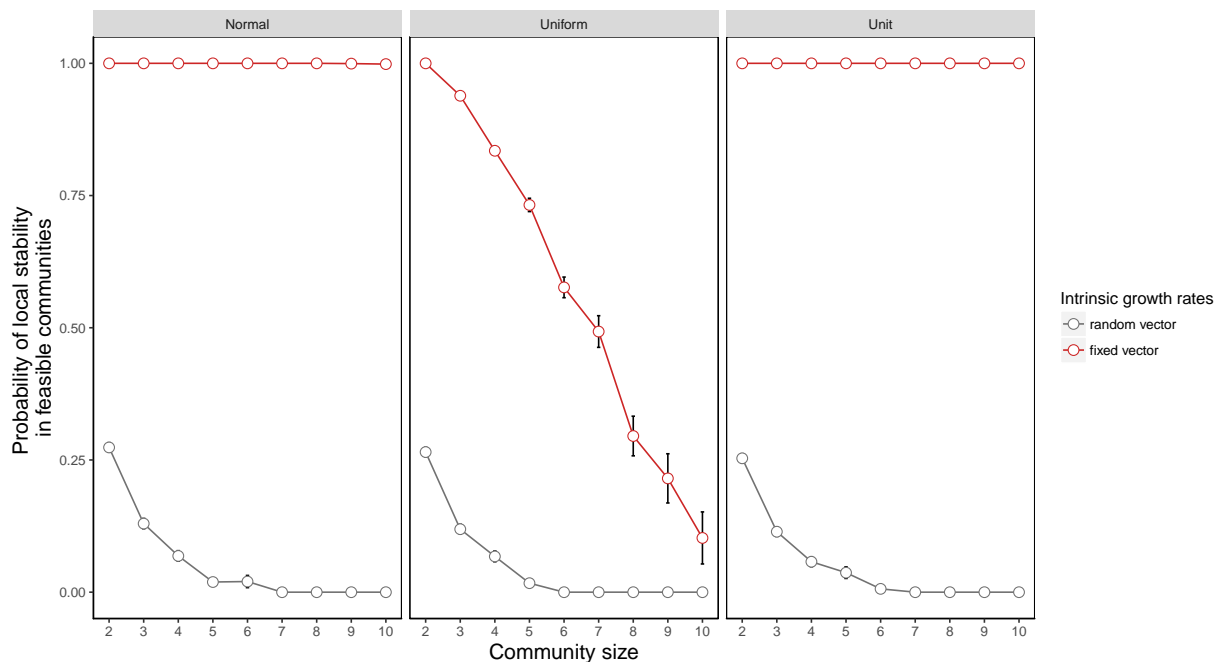


Figure S3: Equivalent to Figure 1a in the manuscript, but here the sampling of the interaction matrix A follows a normal, uniform, and unit distribution, separately. Results change under a uniform distribution.

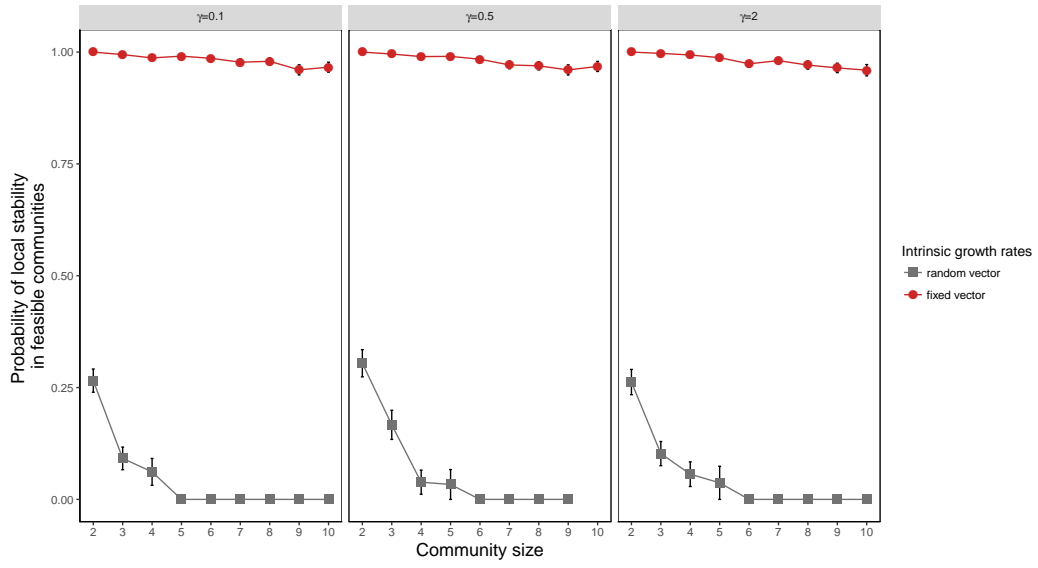


Figure S4: Equivalent to Figure 1a in the manuscript, but here the interaction strength σ is varied such that $\gamma = \sigma\sqrt{SC}$ takes values of 0.1, 0.5 and 2. In general, we found that results are robust to changes to the interaction strength.

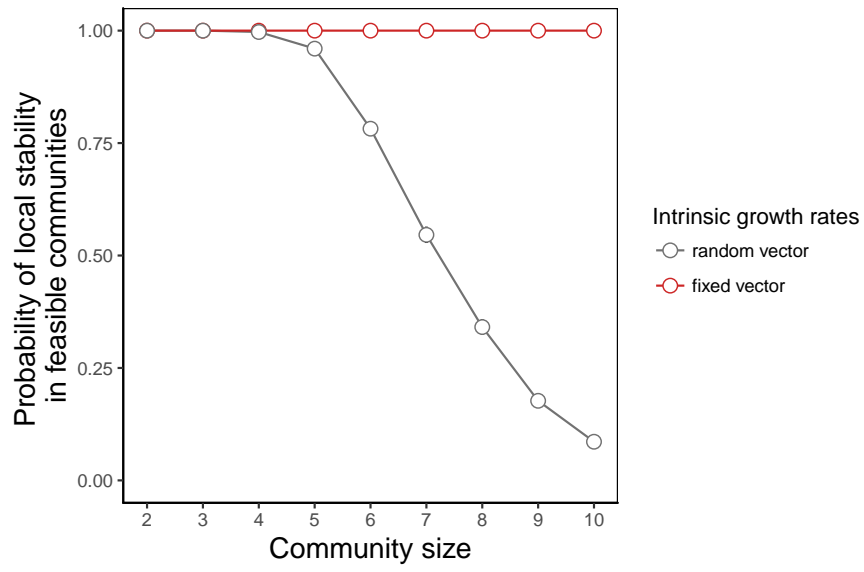


Figure S5: Equivalent to Figure 1a in the manuscript, but here all the diagonal elements are $a_{ii} = -1$.

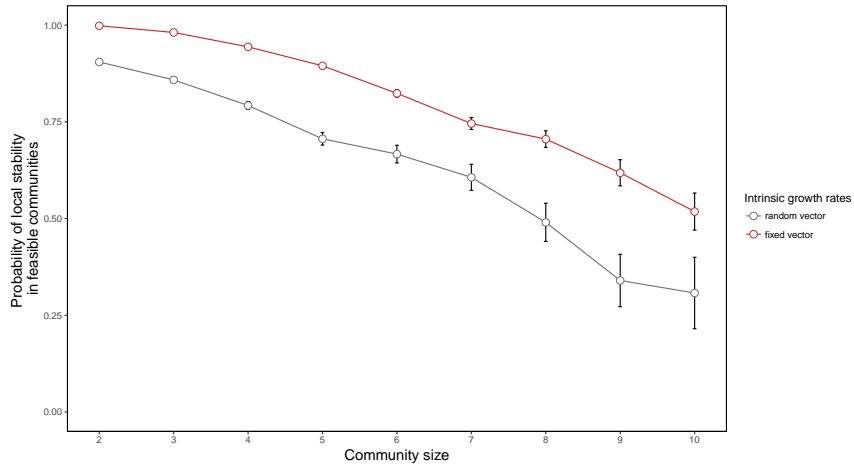


Figure S6: Equivalent to Figure 1a in the manuscript, but here the fraction of negative diagonal elements in the interaction matrix A is reduced to 0.3 (originally it was 0.7). Results change under variations to the diagonal elements.

S3 Robustness of the feasibility-stability relationship to different parameterizations

As shown in Figure 3 in the manuscript, sampling intrinsic growth rates inside the feasible parameter space can significantly increase the probability of local stability of a community. Here, we tested the robustness of this result against a different parameterization of sampling distribution of the interaction matrix A (Figure S7).

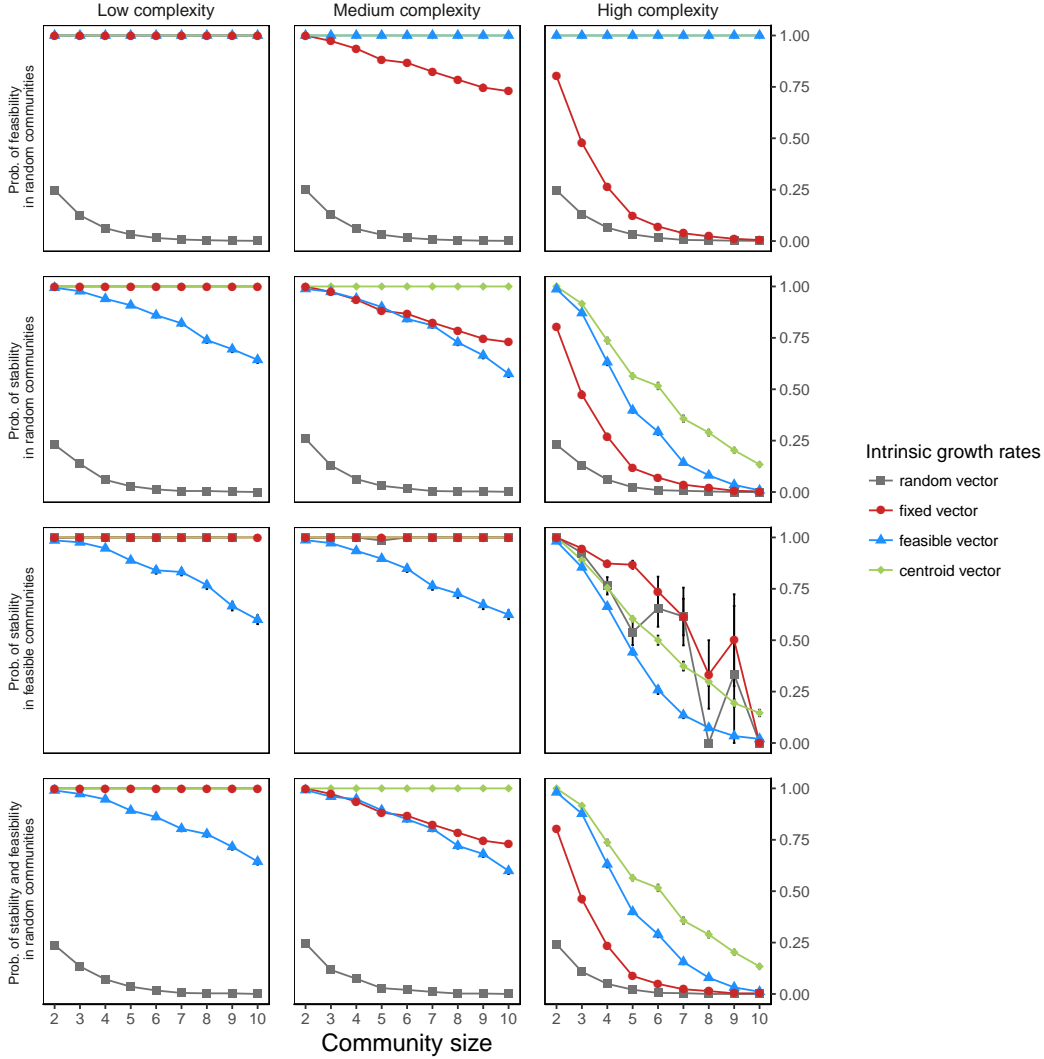


Figure S7: Equivalent to Figure 3 and Figure 4 in the manuscript, but here the sampling distribution of the matrix A follows a uniform distribution.

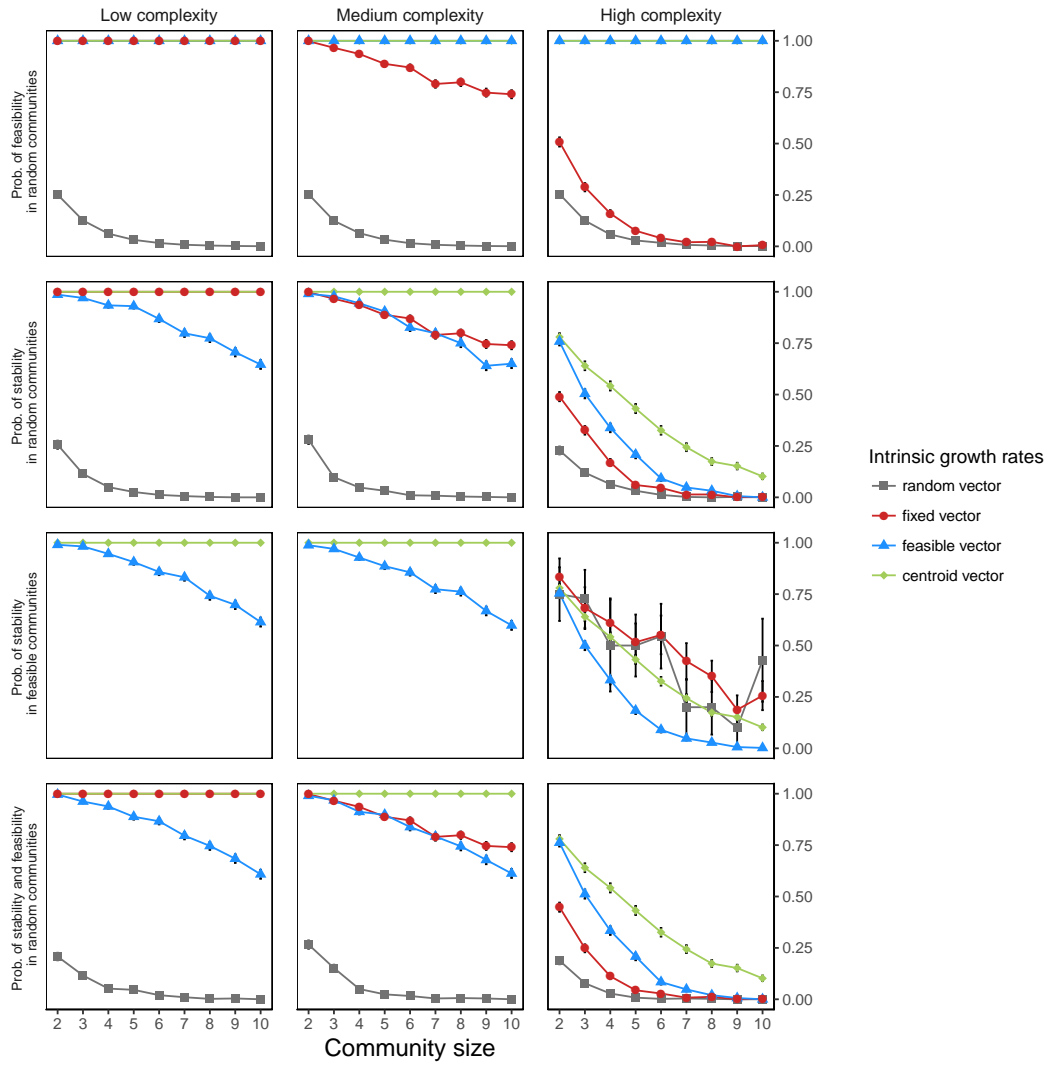


Figure S8: Equivalent to Figure 3 and Figure 4 in the manuscript, but here the connectance C is set to 1.

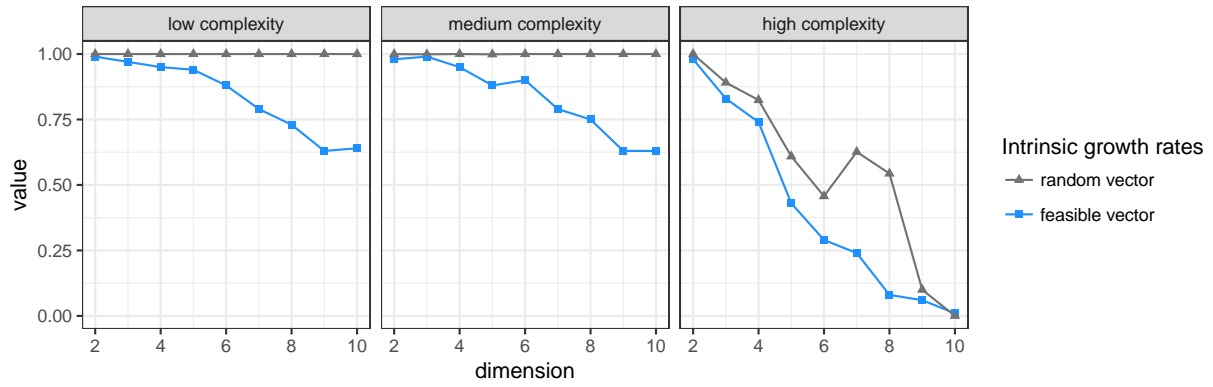


Figure S9: Equivalent to the Fig. 4(a) in the manuscript, but here the random vector is generated from a standard normal distribution $N(0, 1)$.

S4 Stabilizing effect of negative diagonal values in the interaction matrix

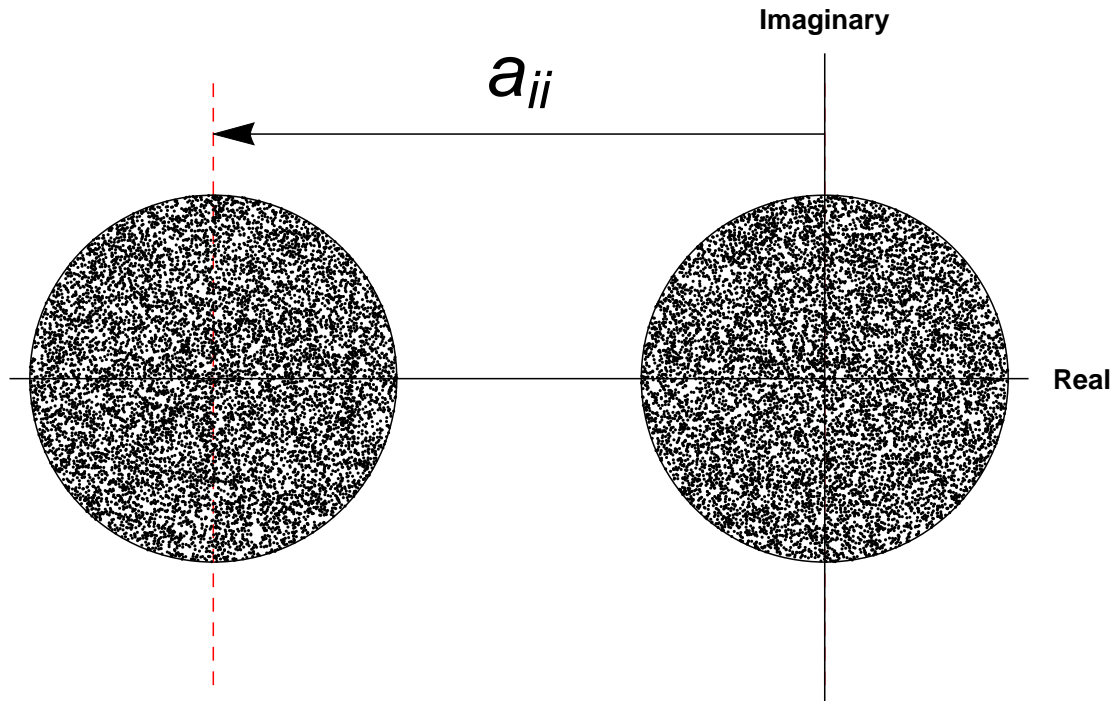


Figure S10: Cartoon representing the stabilizing effect of the negative diagonal values in the interaction matrix. This cartoon shows, using Gershgorin discs (each point inside the discs correspond to an eigenvalue associated with a fictitious random interaction matrix and equilibrium point) (McCann, 2011), that the more negative the diagonal values a_{ii} in the interaction matrix \mathbf{A} (moving towards the left of the Real numbers on the x-axis), the more negative the eigenvalues derived from the interaction matrix. Recall that locally stable matrices are characterized by negative real parts of eigenvalues.

S5 Filtering effect of stability on feasible communities

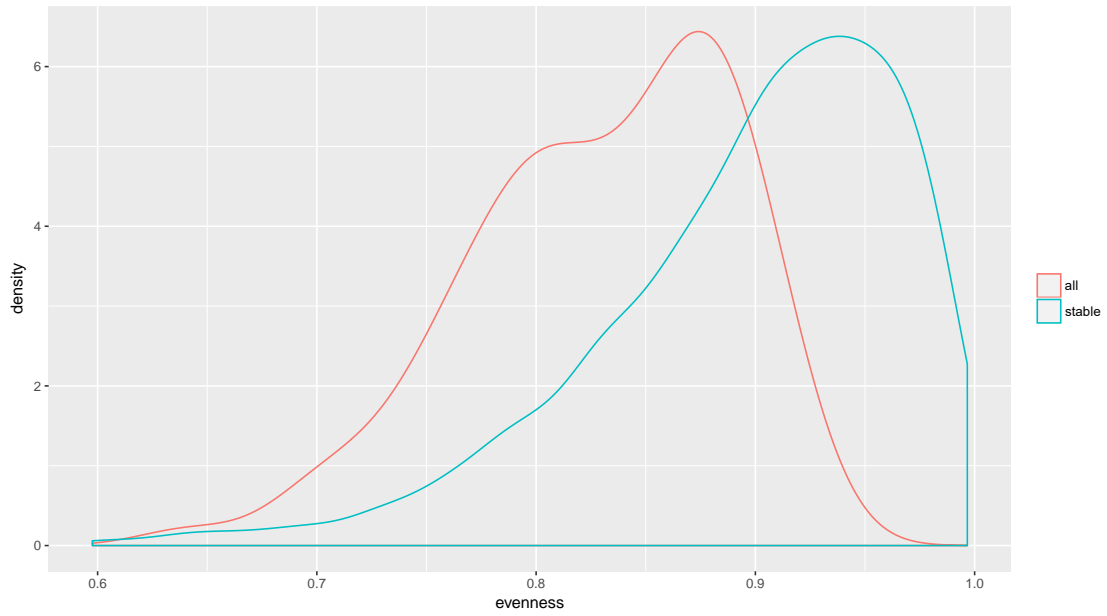


Figure S11: Equivalent to Fig. 5 in the manuscript, except that the abundance distribution is drawn from a uniform distribution $U(0, 1)$.

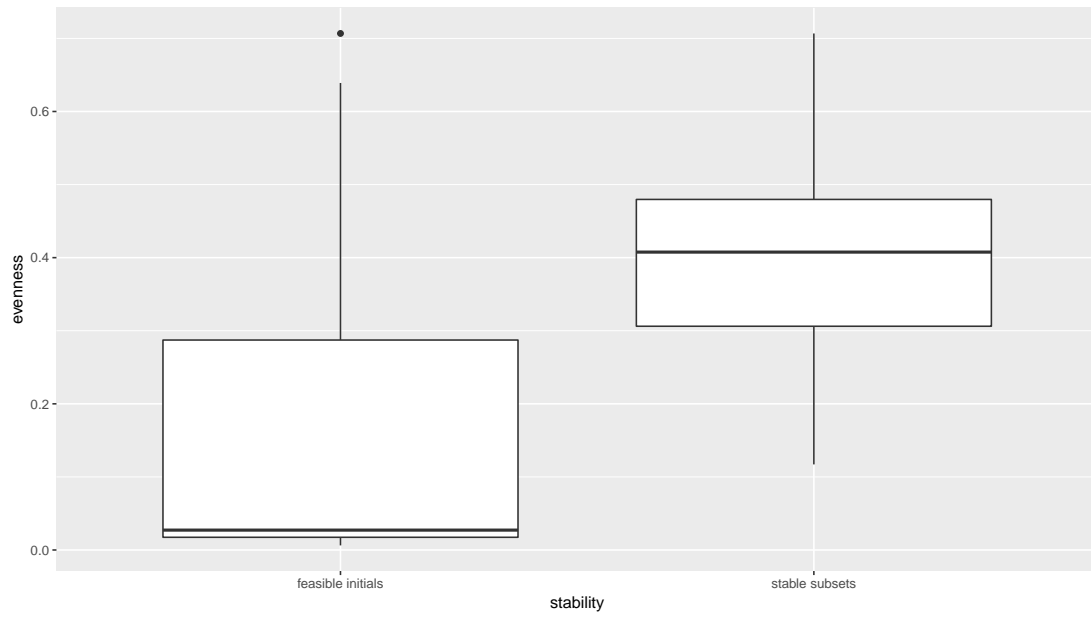


Figure S12: 1000 different interaction matrix are generated with the same set of parameters in Fig. 5 in the manuscript. The box plot of the mean evenness of the initial SAD of the feasible communities and the mean evenness of the stable SAD of the feasible communities are shown here. It is clear that the evenness in the stable communities are higher than that in the set of initial conditions.

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